Scattering of UHECR at small pitch angle by plasma wave turbulence

It is very easy to be modern.

Salvador Dali

1. Relevant observations
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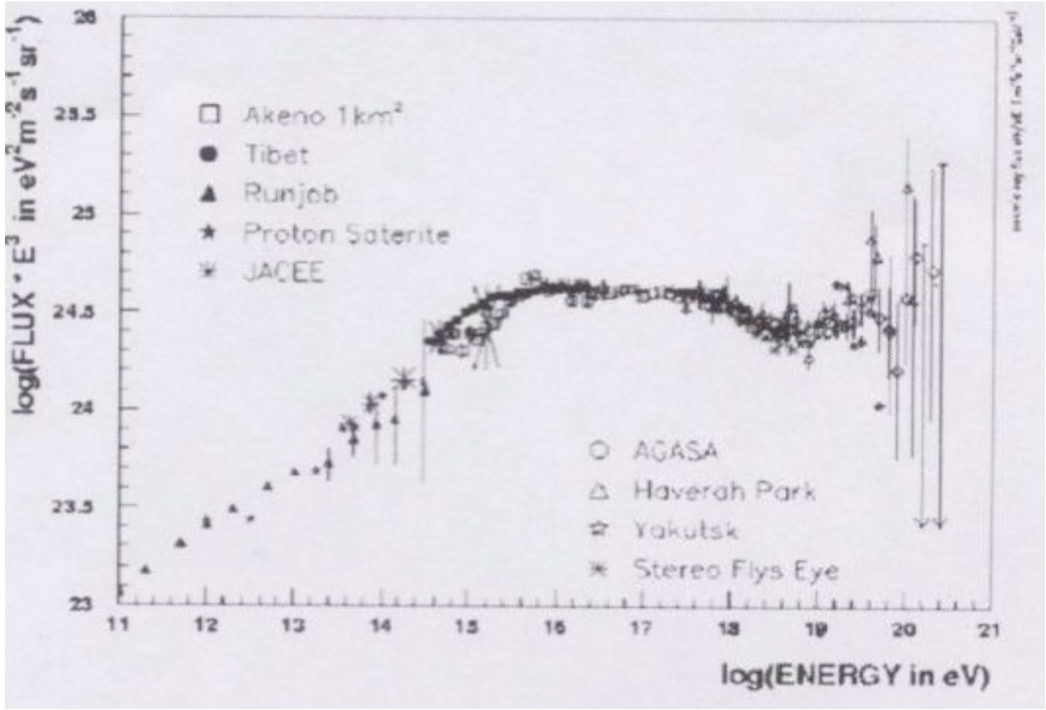
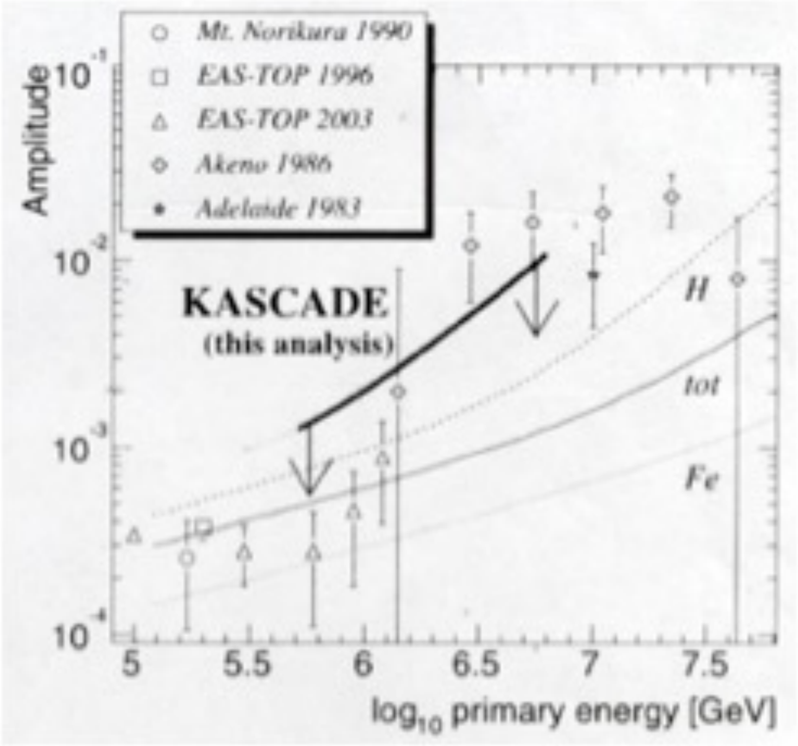
Relevant observations



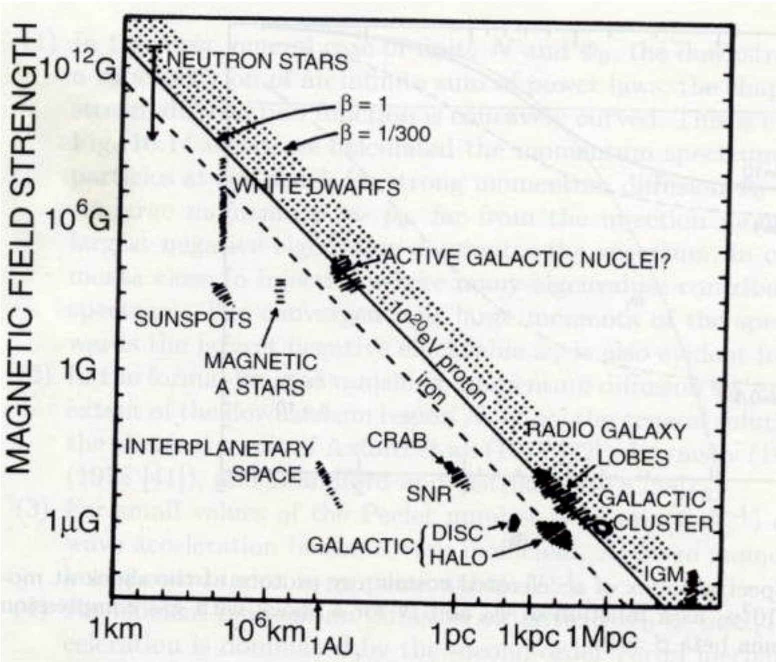


(Antoni et al. 2004)

1) Individual particle energies up to 10^{20} eV for hadrons and 10^{14} eV for electrons (positrons and negatrons) with power law energy distributions $N(E) \propto E^{-s}$ over wide energy ranges. At relativistic energies 100 times more hadrons then electrons.

2) CRs are highly isotropic, although sources are located in galactic plane $\rightarrow$ efficient scattering mechanism required!

Common feature: compact objects with high-velocity (often relativistic) outflows interacting with the ambient medium.
These observations tell us WHERE cosmic rays are accelerated but not necessarily HOW they are accelerated!
Crucial question: How is directed kinetic outflow energy converted into relativistic charged particles (=relativistic plasma beam physics)?



1) Energy density of cosmic ray hadrons

$$w = 4\pi \sum_i \int_0^\infty dE_{\rm kin} E_{\rm kin} N_i(E_{\rm kin}) = 0.5 \text{ eV cm}^{-3}$$

and pressure $P = w/3 \simeq 3 \cdot 10^{-13}$ dyne cm^{-2} comparable to energy densities of interstellar gas, galactic magnetic field and the universal microwave background radiation ("global energy equipartition").

2) Steady luminosity of galactic CRs with volume of galaxy $V \simeq 10^{67}$ cm^3:

$$L = wV/ < T > \simeq 10^{40} \text{erg s}^{-1}$$

Restricts possible CR source energetics.

3) Degree of linear polarization of synchrotron radiation from galactic CR electrons: galactic magnetic field contains large fraction of turbulent component $B = B_0 + \delta B$ with $\delta B \simeq B_0$.

4) Most likely scattering mechanism is pitch-angle scattering by turbulent magnetic field fluctuations. What else? Coulomb scattering much too slow because of low ISM gas density $< n >$.

5) Rigidity ordering: evidence for **electromagnetic** CR acceleration and transport processes in turbulent electromagnetic fields $\vec{E} = \vec{0} + \delta\vec{E}$ (no ordered electric field because of huge ISM conductivity, few exceptions: pulsars, magnetic reconnection)), $\vec{B} = \vec{B}_0 + \delta\vec{B}$.

Faraday's induction law relates rot $\delta\vec{E} = -c^{-1}(\partial\delta\vec{B}/\partial t)$.

Lorentz force:

$$\frac{d\vec{p}}{dt} = q\left[\delta\vec{E} + \frac{\vec{v}\times(\vec{B}_0 + \delta\vec{B})}{c}\right]$$

Acceleration requires turbulent electric fields:

$$\frac{dE_{\rm kin}}{dt} = \frac{c^2}{2E_{\rm kin}}\frac{dp^2}{dt} = \frac{c^2}{E_{\rm kin}}q\vec{p}\cdot\delta\vec{E}$$

with $E_{\rm kin} = \sqrt{p^2c^2 + m^2c^4} - mc^2$.

Equal acceleration rates for charged particles at the same magnetic rigidity $R = p/q$:

$$\frac{dp^2}{dt} = 2q\vec{p}\cdot\delta\vec{E} \quad \rightarrow \quad \frac{dR^2}{dt} = 2\vec{R}\cdot\delta\vec{E}$$

Any electromagnetic acceleration process explains 100 times more hadrons than electrons at relativistic energies $E_{\rm kin} \gg m_pc^2 = 1$ GeV because $m_p = 1836m_e$

6) To a large extent, our progress in understanding CR dynamics in cosmic plasmas depends on our understanding of the magnetic and electric field fluctuations

Here we address two important issues:

- Nature of cosmic electromagnetic fluctuations in magnetized (e.g. ISM) and nonmagnetized (intergalactic medium (IGM)) cosmic plasmas
- Which equations describe the dynamics of cosmic rays for given and specified electromagnetic fields (*test-particle approach*)? And what are the CR transport parameters (diffusion coefficients, convection speeds, acceleration rates) ?

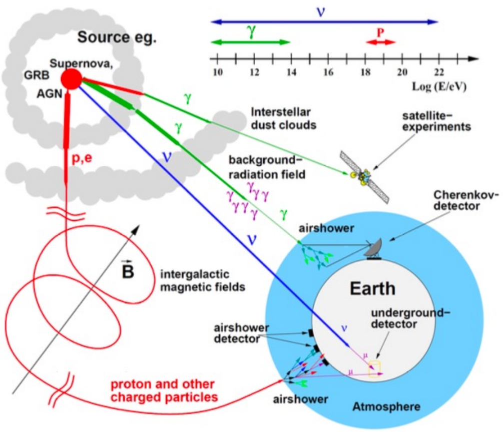


Figure 7: Sketch of cosmic ray life. Courtesy R. Wagner.

Fig. 7 sketches the typical life of a cosmic ray particle: after being accelerated in individual sources such like supernova remnants, active galactic nuclei or gamma-ray bursts, it stochastically propagates in the partially turbulent magnetic field and interacts with the ambient photon and matter fields, generating nonthermal photon and neutrino radiation. In the case of galactic CRs this takes about 10^7 years before detection by near-Earth or ground based detectors.

The electromagnetic fields fulfill Maxwell equations

$$\nabla \times \vec{B}(\vec{x},t) - \frac{1}{c}\frac{\partial}{\partial t}\vec{E}(\vec{x},t) = \frac{4\pi}{c}\sum_a q_a \int d^3p\, \vec{v}\, f_a(\vec{x},\vec{p},t), \tag{2}$$

$$\nabla \cdot \vec{B}(\vec{x},t) = 0, \tag{3}$$

$$\nabla \times \vec{E}(\vec{x},t) + \frac{1}{c}\frac{\partial}{\partial t}\vec{B}(\vec{x},t) = 0. \tag{4}$$

$$\nabla \cdot \vec{E}(\vec{x},t) = 4\pi \sum_a q_a \int d^3p\, f_a(\vec{x},\vec{p},t), \tag{5}$$

where the charged particle's phase space distributions $f_a(\vec{x},\vec{p},t)$ determine the current and charge densities on the right-hand side of Eqs. (2) and (5).

Each phase space distribution $f_a(\vec{x},\vec{p},t)$ of ionized interstellar gas and CR particles fulfills the collisionless Boltzmann equation

$$\frac{\partial f_a}{\partial t} + \vec{v}\cdot\frac{\partial f_a}{\partial \vec{x}} + q_a[\vec{E} + \frac{\vec{v}\times\vec{B}}{c}]\cdot\frac{\partial f_a}{\partial \vec{p}} = Q_a(\vec{x},\vec{p},t), \tag{6}$$

where the source term $Q_a(\vec{x},\vec{p},t)$ accounts for sources and sinks of particles and other (than the Lorentz force) electromagnetic interactions such as radiative reaction and/or the mirror force in nonuniform guide magnetic fields $\vec{B}_0$. Via the Lorentz force the electromagnetic fields determine the behaviour of the particle distribution functions f_a.

Method

Because of the complicated nonlinear equations of motion of charged particles in partially random electromagnetic fields there are only two methods to study theoretically particle acceleration and transport:
(i) numerical simulations of highly idealized configurations,
(ii) quasilinear perturbation theory valid for small turbulence levels $q_L = |\delta\vec{B}| \ll \vec{B}_0$.
Both have their advantages and shortcomings, and they complement each other.

Fokker-Planck equation -- assumptions on the fluctuating electromagnetic field turbulence

- (1) Gaussian statistics,
- (2) adiabatic approximation,
- (3) homogeneous and quasi-stationary turbulence,
- (4) existence of a small enough finite decorrelation time of second-order correlation functions,
- (5) random phase (between particles and fluctuations),

In a moving medium of arbitrary speed $U \parallel \vec{B}_0 \parallel \vec{e}_Z$ the Fokker-Planck equation reads with $\Gamma = [1-(U/c)^2]^{-1/2}$:

$$\Gamma\left[1+\frac{Uv\mu}{c^2}\right]\left[\frac{\partial f_0}{\partial t}-\frac{1}{v}\frac{\partial U}{\partial t}\Gamma^2\left(\mu p\frac{\partial f_0}{\partial p}+(1-\mu^2)\frac{\partial f_0}{\partial \mu}\right)\right]$$
$$+\Gamma\left[U+v\mu\right]\left[\frac{\partial f_0}{\partial Z}-\frac{1}{v}\frac{\partial U}{\partial z}\Gamma^2\left(\mu p\frac{\partial f_0}{\partial p}+(1-\mu^2)\frac{\partial f_0}{\partial \mu}\right)\right]$$
$$+\mathcal{N}f_0+\mathcal{R}f_0-S(\vec{X},p,\mu,t)=p^{-2}\frac{\partial}{\partial x_\alpha}p^2 D_{\alpha\sigma}\frac{\partial f_0}{\partial x_\sigma} \tag{7}$$

The phase space coordinates have to be taken in the mixed comoving coordinate system (time and space coordinates $\vec{x}$ in the laboratory (=observer) system and particle's momentum coordinates p and $\mu = p_\parallel/p$ in the rest frame of the streaming plasma). In Eq. (7) we use the Einstein sum convention for indices, and $x_\alpha \in [\mu, p, X, Y]$ represent the four phase space variables with non-vanishing stochastic fields $\delta\vec{E}$ and $\delta\vec{B}$. Consequently, the term on the right-hand side represents 16 different Fokker-Planck coefficients: but, depending on the turbulent fields considered not all of them are non-zero and some are much larger than others. $S(\vec{X},p,\mu,t)$ represents additional sources and sinks of particles.

In a medium at rest the Fokker-Planck transport equation (7) reduces to

$$\frac{\partial f_0}{\partial t}+v\mu\frac{\partial f_0}{\partial Z}+\mathcal{N}f_0+\mathcal{R}f_0-S(\vec{X},p,\mu,t)=p^{-2}\frac{\partial}{\partial x_\alpha}p^2 D_{\alpha\sigma}\frac{\partial f_0}{\partial x_\sigma}, \tag{10}$$

Magnetized space plasmas contain low-frequency linear ($\delta B \ll B_0$) transverse MHD waves (such as shear Alfven and magnetosonic plasma waves) with dispersion relations $\omega_R^2 = V_A^2 k_\parallel^2$ and $\omega_R^2 = V_A^2 k^2$, respectively. The induction law then indicates for MHD waves $\delta E = (V_A/c)\delta B \ll \delta B$

As we demonstrate next, a perturbation scheme based on $B_0 \gg \delta B \gg \delta E$ corresponds to the reduction

$$< f > (\vec{X},p,\mu,\phi,t) \to f_0(\vec{X},p,\mu,t) \to F(\vec{X},p,t) \tag{11}$$

to gyrotropic $f_0(\vec{X},p,\mu,t)$ and to isotropic, gyrotropic distributions functions $F(\vec{X},p,t)$, respectively, in excellent agreement with the observed isotropy of CRs.

Before proceeding, we estimate the relative strength of the different Fokker-Planck coefficients. With $\epsilon = V_A/v \ll 1$ these scale as

$$D_{\mu\mu}\,(\simeq D_{\phi\phi}) \simeq D_0 = a_1\Omega_p\frac{\delta B^2}{B_0^2} = a_1\Omega_p q_L \ll a_1\Omega_p,$$
$$D_{pp} \simeq D_0\epsilon^2 p^2,\ D_{X,Y} \simeq R_L^2 D_0,$$
$$D_{\mu p}(\simeq D_{\phi p}) \simeq D_0\epsilon p,\ D_{\mu X} \simeq D_{\phi X} = R_L D_0 \tag{12}$$

Consequently, the associated times scales for pitch-angle scattering ($T_\mu \simeq D_{\mu\mu}^{-1}$), momentum diffusion ($T_p \simeq p^2/D_{pp}$) and perpendicular spatial gyro-center diffusion ($T_X \simeq X^2/D_{XX}$) scale as

$$T_\mu \simeq T_\phi = T_0 \simeq D_0^{-1},\ T_p \simeq \frac{T_0}{\epsilon^2} \gg T_0,\ T_X \simeq \frac{X^2}{R_L^2}T_0 \gg T_0 \tag{13}$$

Therefore, in the presence of low-frequency MHD fluctuations the particles will relax most quickly on the time scale $\min[\Omega_p^{-1}, T_0]$ to an isotropic, gyrotropic distribution function, which then on considerably longer time scales T_X and T_p undergoes diffusion in position space and momentum space, respectively.

Moreover, spatial convection and momentum convection is possible stemming from the mixed Fokker-Planck coefficients such as $D_{\mu X}$ and $D_{\mu p}$.

Diffusion approximation

Our earlier qualitative estimate of Fokker-Planck coefficients for energetic particles with $v \gg V_A$ indicated that the pitch angle Fokker-Planck coefficient $D_{\mu\mu}$ is the largest one. We therefore make the basic assumption of diffusion theory that the gyrotropic particle distribution function $f_0(\vec{X}, p, \mu, t)$ under the action of low-frequency magnetohydrodynamic waves adjusts very quickly to a distribution function through pitch-angle diffusion which is close to the isotropic distribution in the rest frame of the moving background plasma. Defining the isotropic part of the phase space density $F(\vec{X}, z, p, t)$ as the μ-averaged phase space density

$$F(\vec{X}, p, t) \equiv \frac{1}{2} \int_{-1}^{1} d\mu \; f_0(\vec{X}, p, \mu, t), \tag{14}$$

we follow the analysis of Jokipii (1966) and Hasselmann and Wibberenz (1968) to split the total density f_0 into the isotropic part F and an anisotropic part g,

$$f_0(\vec{X}, p, \mu, t) = F(\vec{X}, p, t) + \; g(\vec{X}, p, \mu, t) \tag{15}$$

where because of Eq. (14)

$$\int_{-1}^{1} d\mu \, g(\vec{X}, p, \mu, t) = 0 \tag{16}$$

Singleing out the phase space variable μ by introducing the reduced set of variables $z_{\alpha,\sigma} \in [X, Y, p]$ the Larmor-phase averaged Fokker-Planck coefficients transport equation (7) reads

$$\begin{aligned}
&\Gamma \left[1 + \frac{Uv\mu}{c^2}\right] \left[\frac{\partial f_0}{\partial t} - \frac{1}{v}\frac{\partial U}{\partial t}\Gamma^2 \left(\mu p \frac{\partial f_0}{\partial p} + (1-\mu^2)\frac{\partial f_0}{\partial \mu}\right)\right] \\
&+\Gamma \left[U + v\mu\right] \left(\frac{\partial f_0}{\partial Z} - \frac{1}{v}\frac{\partial U}{\partial z}\Gamma^2 \left(\mu p \frac{\partial f_0}{\partial p} + (1-\mu^2)\frac{\partial f_0}{\partial \mu}\right)\right] \\
&+\mathcal{N} f_0 + \mathcal{R} f_0 - S(\vec{X}, p, \mu, t) = \\
&p^{-2}\frac{\partial}{\partial z_\alpha} p^2 D_{\alpha\sigma} \frac{\partial f_0}{\partial z_\sigma} + \frac{\partial}{\partial \mu} D_{\mu\mu} \frac{\partial f_0}{\partial \mu} + \frac{\partial}{\partial \mu} D_{\mu\sigma} \frac{\partial f_0}{\partial z_\sigma} + p^{-2}\frac{\partial}{\partial z_\alpha} p^2 D_{\alpha\mu} \frac{\partial f_0}{\partial \mu},
\end{aligned} \tag{17}$$

Instead of manipulating this complicated equation, we follow the historical development and consider simplified versions of the full transport equation.

Full transport equation

We now give up on the magnetostatic approximation and include finite electric field effects. The diffusion approximation of the Fokker-Planck equation (10) leads to

$$\frac{\partial F}{\partial t} + \mathcal{R}F - S(\vec{X},p,t) - \begin{pmatrix} V_X \\ V_Y \\ V_Z \\ V_p \end{pmatrix} \cdot \begin{pmatrix} \partial_X F \\ \partial_Y F \\ \partial_z F \\ p^{-2}\partial_p p^2 F \end{pmatrix}$$

$$= \begin{pmatrix} \partial_X \\ \partial_Y \\ \partial_z \\ p^{-2}\partial_p p^2 \end{pmatrix} \cdot \begin{pmatrix} \kappa_{XX} & \kappa_{XY} & \kappa_{XZ} & \kappa_{Xp} \\ \kappa_{YX} & \kappa_{YY} & \kappa_{YZ} & \kappa_{Yp} \\ \kappa_{ZX} & \kappa_{ZY} & \kappa_{ZZ} & \kappa_{Zp} \\ \kappa_{pX} & \kappa_{pY} & \kappa_{pZ} & \kappa_{pp} \end{pmatrix} \begin{pmatrix} \partial_X F \\ \partial_Y F \\ \partial_z F \\ p^{-2}\partial_p p^2 F \end{pmatrix} \tag{40}$$

We identify some of the individual 20 pitch-angle averaged diffusion coefficients and convection speeds as

$$\kappa_{ZZ} = \frac{v\lambda_\parallel}{3} = \frac{v^2}{8}\int_{-1}^{1} d\mu \frac{(1-\mu^2)^2}{D_{\mu\mu}} \propto \int_{-1}^{1} d\mu \frac{1}{D_{\mu\mu}}!, \tag{41}$$

$$\kappa_{XX} = \frac{1}{2}\int_{-1}^{1} d\mu \left[D_{XX} - \frac{D_{X\mu}D_{\mu X}}{D_{\mu\mu}}\right]$$
$$+\frac{sign(q_a)vR_L}{12L_2}\left[\int_{-1}^{1} d\mu \frac{\mu(1-\mu^2)D_{X\mu}}{D_{\mu\mu}} - \int_{-1}^{1} d\mu \frac{(1-\mu^3)D_{\mu X}}{D_{\mu\mu}}\right]$$
$$-\frac{v^2R_L^2}{72L_2^2}\left[\int_{-1}^{1} d\mu \frac{\mu(1-\mu^2)(1-\mu^3)}{D_{\mu\mu}}\right.$$
$$\left.+\frac{U}{v}\int_{-1}^{1} d\mu \frac{\mu(1-\mu^2)^2}{D_{\mu\mu}} + \frac{2}{3}\int_{-1}^{1} d\mu \frac{\mu(1-\mu^2)(1-\mu^3)}{D_{\mu\mu}}\right], \tag{42}$$

$$\kappa_{Yp} = \frac{1}{2}\int_{-1}^{1} d\mu \left[D_{Yp} - \frac{D_{Y\mu}D_{\mu p}}{D_{\mu\mu}}\right] + \frac{sign(q_a)vR_L}{12L_1}\int_{-1}^{1} d\mu \frac{(1-\mu)^3 D_{\mu p}}{D_{\mu\mu}}, \tag{43}$$

$$\kappa_{Zp} = \frac{v}{4}\int_{-1}^{1} d\mu \frac{(1-\mu^2)D_{\mu p}}{D_{\mu\mu}}, \tag{44}$$

$$\kappa_{pp} = \frac{1}{2}\int_{-1}^{1} d\mu \left[D_{pp} - \frac{D_{\mu p}D_{p\mu}}{D_{\mu\mu}}\right]$$
$$-\frac{p}{4}\frac{\partial U}{\partial z}\left[\frac{U}{v}\int_{-1}^{1} d\mu \frac{1-\mu^2)D_{\mu p}}{D_{\mu\mu}} + \frac{2}{3}\int_{-1}^{1} d\mu \frac{(1-\mu^3)D_{\mu p}}{D_{\mu\mu}}\right] \tag{45}$$

The convection speeds are given by

$$V_X = \frac{\kappa_{ZX}}{L_3} + \frac{\gamma+1}{\gamma v^2}U\frac{\partial U}{\partial z}\kappa_{ZX} \tag{46}$$

$$V_Y = \frac{\kappa_{ZY}}{L_3} + \frac{\gamma+1}{\gamma v^2}U\frac{\partial U}{\partial z}\kappa_{ZY} \tag{47}$$

$$V_Z = -U + \frac{\kappa_{ZZ}}{L_3} + \frac{\gamma+1}{2\gamma v^2}U\frac{\partial U}{\partial z}\kappa_{zz} \tag{48}$$

$$V_p = \frac{1}{3}\frac{\partial U}{\partial z}p + \frac{\kappa_{Zp}}{L_3} + \frac{\gamma+1}{\gamma v^2}U\frac{\partial U}{\partial z}\kappa_{Zp} \tag{49}$$

In its general form the diffusion–convection transport equation (40) contains spatial diffusion and spatial convection terms as well as momentum diffusion and momentum convection terms. Since the pioneering work of Fermi (1949, 1954) it has become customary to refer to the latter two as Fermi acceleration of second and first order, respectively.

Pitch angle Fokker-Planck coefficient

With its 20 different diffusion coefficients and 4 convection speeds the general diffusion-convection transport eequation (40) is rather complicated and involved. One has to emphasize that, depending on the type of turbulent electromagnetic fields considered, not all of these 24 CR transport parameters have nonzero values, and some of the transport parameters have much higher values than others, so that simplified versions of the general transport equation (40) are justified.

Most crucial is the knowledge of the pitch-angle Fokker-Planck coefficient $D_{\mu\mu}(\mu)$! Smallest values for κ_{ZZ}, largest values for κ_{pp}

In the ISM the presence of isotropically distributed fast magnetosonic waves is essential, as these fluctuations allow transit-time damping interactions with the CRs, which provide the dominating contribution to scattering.

$$g_\mu = \dot{\mu} = \frac{\Omega\sqrt{1-\mu^2}}{B_0}\left[\frac{c}{v}\sqrt{1-\mu^2}\delta E_{||} + \frac{i}{\sqrt{2}}\left[e^{i\phi}(\delta B_r + i\mu\frac{c}{v}\delta E_r) - e^{-i\phi}(\delta B_l - i\mu\frac{c}{v}\delta E_l)\right]\right]$$ gyrofrequency $\Omega = ZeB_0/(mc\gamma)$

$$D_{\mu\mu} = \lim_{t\to\infty}\frac{<(\Delta\mu)^2>}{2t}$$

Step 1. quasilinear approximation - integration over unperturbed orbits

The quasilinear approximation is achieved by replacing in the Fourier transform of the fluctuating electric and magnetic field

$$\delta \mathbf{E}(\mathbf{x}(t), t) = \int_{-\infty}^{+\infty} d^3k \delta \mathbf{E}(\mathbf{k}, t) e^{i\mathbf{k}\cdot\mathbf{x}(t)} \simeq \int_{-\infty}^{+\infty} d^3k \delta \mathbf{E}(\mathbf{k}, t) e^{i\mathbf{k}\cdot\mathbf{x}^0(t)} \tag{71}$$

$$\delta \mathbf{B}(\mathbf{x}(t), t) = \int_{-\infty}^{+\infty} d^3k \delta \mathbf{B}(\mathbf{k}, t) e^{i\mathbf{k}\cdot\mathbf{x}(t)} \simeq \int_{-\infty}^{+\infty} d^3k \delta \mathbf{B}(\mathbf{k}, t) e^{i\mathbf{k}\cdot\mathbf{x}^0(t)} \tag{72}$$

the true particle orbit $\mathbf{x}(t)$ by the unperturbed orbit $\mathbf{x}^0(t)$, resulting in

$$\phi(t) = \phi_0 - \Omega t \tag{73}$$

$$\delta B_{l,r,\|} \approx \sum_{n=-\infty}^{+\infty} d^3k \delta B_{l,r,\|}(\mathbf{k}, t) J_n(W) e^{in(\psi-\phi_0)+i(k_\| v_\| + n\Omega)t + i\mathbf{k}\cdot\mathbf{x}_0}, \tag{74}$$

respectively, where $\mathbf{x}_0 = (x_0, y_0, z_0)$ denotes the initial $(t = 0)$ position of the cosmic ray particle and $W = \frac{v}{|\Omega|} \cdot k_\perp \sqrt{1-\mu^2} = R_L \cdot k_\perp \sqrt{1-\mu^2}$ involving the cosmic ray Larmor radius $R_L = v/|\Omega|$. For the wave vector $\mathbf{k}$ we have used cylindrical coordinates:

$$k_\| = k_z, \quad k_\perp = \sqrt{k_x^2 + k_y^2}, \quad \psi = \mathrm{arccot}(\frac{\mathrm{k_x}}{\mathrm{k_y}}). \tag{75}$$

$$\frac{d\mu}{dt} \simeq h_\mu(t) = \frac{\Omega\sqrt{1-\mu^2}}{B_0} \sum_{n=-\infty}^{\infty} \int d^3k e^{in(\psi-\phi_0)+i(k_\| v_\| + n\Omega)t + i\mathbf{k}\cdot\mathbf{x}_0} \Big(\frac{c}{v}\sqrt{1-\mu^2} J_n(W) \delta E_\|(\mathbf{k}, t)$$
$$+ \frac{i}{\sqrt{2}} J_{n+1}(W) e^{i\psi} [\delta B_r(\mathbf{k}, t) + i\mu \frac{c}{v} \delta E_r(\mathbf{k}, t)] - \frac{i}{\sqrt{2}} J_{n-1}(W) e^{-i\psi} [\delta B_l(\mathbf{k}, t) - i\mu \frac{c}{v} \delta E_l(\mathbf{k}, t)] \Big) \tag{76}$$

Step 2. Quasi-stationary turbulence

As second assumption (after the quasilinear approximation) we use the *quasi-stationary turbulence condition* that the correlation function $< h_\mu(t_1) h^*_\mu(\tau+t_1) >$ depends only on the absolute value of the time difference $[t_2 - t_1| = |\tau|$ so that

$$< h_\mu(t_1) h^*_\mu(\tau + t_1) >=< h_\mu(0) h^*_\mu(0 + \tau) > \tag{81}$$

$$< (\Delta\mu)^2 >= 2 \int_0^t dt_1 \int_{-t_1}^0 d\tau < h_\mu(0) h^*_\mu(0+\tau) >= 2 \int_0^t dt_1 \int_0^{t_1} ds < h_\mu(0) h^*_\mu(s) > \tag{82}$$

Thirdly, we assume that there exists a finite correlation time t_c such that the correlation function $< h_\mu(0) h^*_\mu(s) >\to 0$ falls to a negligible magnitude for $s \to \infty$. This allows us to replace the upper integration boundary in the second integral by infinity so that

$$< (\Delta\mu)^2 >\simeq 2 \int_0^t dt_1 \int_0^\infty ds < h_\mu(0) h^*_\mu(s) >= 2t \int_0^\infty ds < h_\mu(0) h^*_\mu(s) > \tag{83}$$

As can be seen, the two assumptions of quasi-stationary turbulence and the existence of a finite correlation time t_c guarantee diffusive behaviour of quasilinear transport in agreement with the conclusion of Shalchi & Schlickeiser (2004). For the quasilinear Fokker-Planck coefficients (70) we then obtain

$$D_{\mu\mu} = \int_0^\infty ds < h_\mu(0) h^*_\mu(s) > \tag{84}$$

Step 3. homogenous turbulence

$$\frac{1}{(2\pi)^3}\int_{-\infty}^{+\infty} d^3\mathbf{x}_0 e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{x}_0} = \delta(\mathbf{k}-\mathbf{k}') \tag{88}$$

implying that turbulence fields at different wavevectors are uncorrelated. The respective ensemble average of (87) then involve the correlelation tensors

$$< \delta B_\alpha(\mathbf{k},0)\delta B^*_\beta(\mathbf{k}',s) >= \delta(\mathbf{k}-\mathbf{k}')P_{\alpha\beta}(\mathbf{k},s) \tag{89}$$

$$< \delta E_\alpha(\mathbf{k},0)\delta E^*_\beta(\mathbf{k}',s) >= \delta(\mathbf{k}-\mathbf{k}')R_{\alpha\beta}(\mathbf{k},s) \tag{90}$$

$$< \delta B_\alpha(\mathbf{k},0)\delta E^*_\beta(\mathbf{k}',s) >= \delta(\mathbf{k}-\mathbf{k}')T_{\alpha\beta}(\mathbf{k},s) \tag{91}$$

$$< \delta E_\alpha(\mathbf{k},0)\delta B^*_\beta(\mathbf{k}',s) >= \delta(\mathbf{k}-\mathbf{k}')Q_{\alpha\beta}(\mathbf{k},s). \tag{92}$$

Step 4. stochastic phase approximation – that the initial phase ϕ_0 of the cosmic particle is a random variable that can take on any value between 0 and 2π, the averaging over ϕ_0 results in

Step 5. plasma wave turbulence – time integration

We follow the approach for the electromagnetic turbulence that represents the Fourier transforms of the magnetic and electric field fluctuations as superposition of N individual weakly damped plasma modes of frequencies

$$\omega = \omega_j(\mathbf{k}) = \omega_{R,j}(\mathbf{k}) - i\gamma_j(\mathbf{k}), \tag{94}$$

$j = 1, ...N$, which can have both the real and imaginary parts with $|\gamma_j| << |\omega_{R,j}|$, so that

$$[\mathbf{B}(\mathbf{k},t), \mathbf{E}(\mathbf{k},t)] = \sum_{j=1}^{N} \left[\mathbf{B}^j(\mathbf{k}), \mathbf{E}^j(\mathbf{k})\right] e^{-i\omega_j t} \tag{95}$$

Damping of the waves is counted with the a positive $\gamma^j > 0$. We need to add Maxwell's induction law

$$\mathbf{B}^j(\mathbf{k}) = \frac{c}{\omega_{R,j}} \mathbf{k} \times \mathbf{E}^j(\mathbf{k}). \tag{96}$$

As a consequence of (95), the magnetic correlation tensor (89) becomes

$$P_{\alpha\beta}(\mathbf{k}, s) = \sum_{j=1}^{N} P^j_{\alpha\beta}(\mathbf{k}) e^{-i\omega_{R,j}(\mathbf{k})s - \gamma_j(\mathbf{k})s}, \tag{97}$$

where

$$P^j_{\alpha\beta}(\mathbf{k}) = < B^j_\alpha(\mathbf{k}) B^{j*}_\beta(\mathbf{k}_1) > \delta(\mathbf{k} - \mathbf{k}_1). \tag{98}$$

Corresponding relations hold for the other three correlation tensors.

$$\mathcal{R}_j(\gamma_j) = \int_0^\infty ds\, e^{-i(k_\parallel v_\parallel + \omega_{R,j} + n\Omega)s - \gamma_j s} = \frac{\gamma_j(\mathbf{k})}{\gamma_j^2(\mathbf{k}) + [k_\parallel v_\parallel + \omega_{R,j}(\mathbf{k}) + n\Omega)]^2} \tag{100}$$

In the case of negligible damping $\gamma \to 0$, use of the δ-function representation

$$\lim_{\gamma -> 0} \frac{\gamma}{\gamma^2 + \xi^2} = \pi\delta(\xi) \tag{101}$$

reduces the resonance function (100) to sharp δ-functions

$$\mathbf{R}^j(\gamma = 0) = \pi\delta(k_\parallel v_\parallel + \omega_{R,j} + n\Omega)).$$

Step 6. turbulence geometry

It remains to specify the geometry of the plasma wave turbulence through the correlation tensors which, according to Mattheaus & Smith (1981), have the form

$$P^j_{\alpha\beta}(\mathbf{k}) = \frac{g^j_i(\mathbf{k})}{k^2}\left[\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2} + i\sigma(\mathbf{k})\epsilon_{\alpha\beta\lambda}\frac{k_\lambda}{k}\right], \tag{104}$$

where $\sigma(\mathbf{k})$ is the magnetic helicity and the function $g(\mathbf{k})$ determines different turbulence geometries. This will be discussed in Sec. 5.2.

$$\begin{aligned} D_{\mu\mu} = \frac{\Omega^2}{B_0^2}(1-\mu^2)\sum_{j=1}^{N}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty} d^3k \frac{\gamma_j(\mathbf{k})}{\gamma_j^2(\mathbf{k}) + [k_\parallel v_\parallel + \omega_{R,j}(\mathbf{k}) + n\Omega)]^2} \\ \cdot\Big(\frac{c^2}{v^2}(1-\mu^2)J_n^2(W)R^j_{\|}(\mathbf{k},s) + \frac{1}{2}J^2_{n+1}(W)[P^j_{RR}(\mathbf{k},s)) + \mu^2\frac{c^2}{v^2}R^j_{RR}(\mathbf{k},s) + i\mu\frac{c}{v}(Q^j_{RR}(\mathbf{k},s) - T^j_{RR}(\mathbf{k},s))] \\ +\frac{1}{2}J^2_{n-1}(W)[P^j_{LL}(\mathbf{k},s) + \mu^2\frac{c^2}{v^2}R^j_{LL}(\mathbf{k},s) + i\mu\frac{c}{v}(T^j_{LL}(\mathbf{k},s) - Q^j_{LL}(\mathbf{k},s))] \\ -\frac{1}{2}J_{n-1}(W)J_{n+1}(W)[e^{2i\psi}(P^j_{RL}(\mathbf{k},s) - \mu^2\frac{c^2}{v^2}R^j_{RL}(\mathbf{k},s) + i\mu\frac{c}{v}(T^j_{RL}(\mathbf{k},s) + Q^j_{RL}(\mathbf{k},s))) \\ +e^{-2i\psi}(P^j_{LR}(\mathbf{k},s) - \mu^2\frac{c^2}{v^2}R^j_{LR}(\mathbf{k},s)) - i\mu\frac{c}{v}(T^j_{LR}(\mathbf{k},s) + Q^j_{LR}(\mathbf{k},s)))] \\ +\frac{ic\sqrt{1-\mu^2}}{\sqrt{2}v}J_n(W)[J_{n+1}(W)(e^{i\psi}T^j_{R|}(\mathbf{k},s) - e^{-i\psi}Q^j_{|R}(\mathbf{k},s) + i\mu\frac{c}{v}(R^j_{R|}(\mathbf{k},s)e^{i\psi} + R^j_{|R}(\mathbf{k},s)e^{-i\psi}) \\ +J_{n-1}(W)(e^{i\psi}Q^j_{|L}(\mathbf{k},s) - e^{-i\psi}T^j_{L|}(\mathbf{k},s) + i\mu\frac{c}{v}(R^j_{|L}(\mathbf{k},s)e^{i\psi} + R^j_{L|}(\mathbf{k},s)e^{-i\psi}))\Big] \end{aligned} \tag{103}$$

Step 7: isotropic turbulence

Throughout this work we consider isotropic linearly polarised magnetohydrodynamic turbulence so that the components of the magnetic turbulence tensor for plasma mode j is

$$P^j_{\alpha\beta}(\vec{k}) = \frac{g^j(\vec{k})}{8\pi k^2}(\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2}). \tag{145}$$

The magetic energy density in wave component j then is

$$(\delta B)^2_j = \int d^3k \sum_{i=1}^{3} P_{ii}(\vec{k}) = \int_0^\infty dk g^j(k) \tag{146}$$

We also adopt a Kolmogorov-like power law dependence (index $q > 1$) of $g^j(k)$ above the minimum wavenumber k_{min}

$$g^j(k) = g^j_0 k^{-q} \quad \text{for } k > k_{min}. \tag{147}$$

The normalisation (147) then implies

$$g^j_0 = (q-1)(\delta B)^2_j k^{q-1}_{min}. \tag{148}$$

Moreover we adopt a vanishing cross helicity of each plasma mode, i.e. equal intensity of forward and backward moving waves, so that g^j_0 refers to the total energy density of each mode.

Step 8: Dispersion relation

(1) incompressional *shear Alfven waves* with dispersion relation

$$\omega_R^2 = V_A^2 k_\parallel^2 \tag{129}$$

at parallel wavenumbers $|k_\parallel| \ll \Omega_{p,0}/V_A$, which have no magnetic field component along the ordered background magnetic field $\delta B_z \; (\parallel \vec{B}_0) = 0$,

(2) the *fast magnetosonic waves* with dispersion relation

$$\omega_R^2 = V_A^2 k^2, \;\; k^2 = k_\parallel^2 + k_\perp^2 \tag{130}$$

for wavenumbers $|k| \ll \Omega_{p,0}/V_A$, which have a compressive magnetic field component $\delta B_z \neq 0$ for oblique propagation angles $\theta = \arccos^{-1}(k_\parallel/k) \neq 0$.

Schlickeiser and Miller (1998) investigated the quasilinear interactions of charged particles with these two plasma waves. In case of negligible wave damping the interactions are of resonant nature: a cosmic ray particle of given velocity v, pitch angle cosine μ and gyrofrequency $\Omega_c = \Omega_{c,0}/\gamma$ interacts with waves whose wavenumber and real frequencies obey the condition

$$\boxed{\omega_R(k) = v\mu k_\parallel + n\Omega_c,} \tag{131}$$

for entire $n = 0, \pm 1, \pm 2, \ldots$.

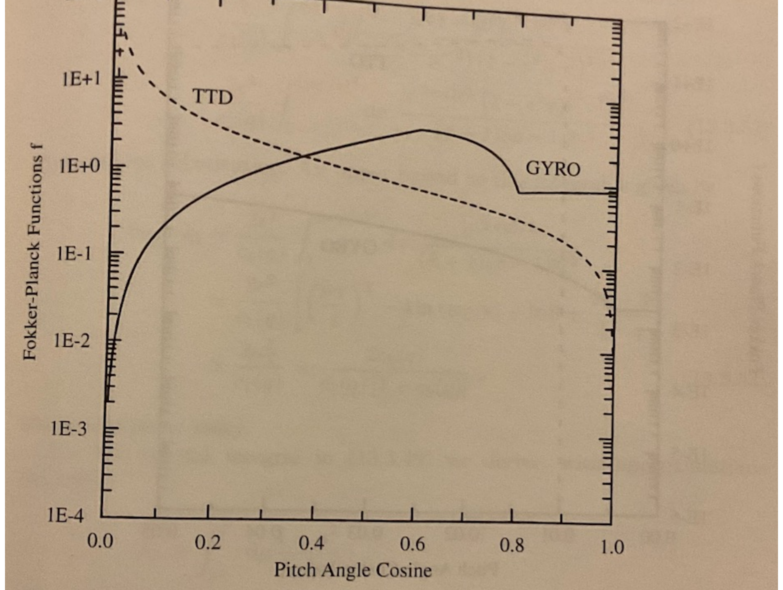


Fig. 13.17. Variation of the Fokker-Planck gyroresonance function f_G (**solid curve**) transit-time damping function f_T (**dashed curve**) as a function of the cosine of pitch angle $0 \le \mu \le 1$ calculated for values of $\epsilon = V_A/v = 0.01$ and $q = 5/3$

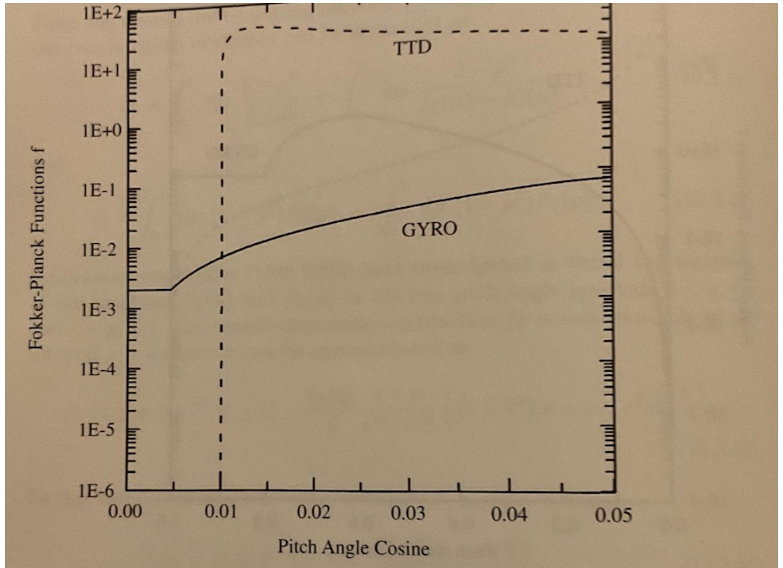


Fig. 13.18. Variation of the Fokker-Planck gyroresonance function f_G (**solid curve**) transit-time damping function f_T (**dashed curve**) as a function in the cosine of pitch angle interval $0 \le \mu \le 0.05$ calculated for values of $\epsilon = V_A/v = 0.01$ and $q = 1.5$. One clearly notices that transit-time damping does not contribute to the scattering of particles with pitch angles $|\mu| < \epsilon = 0.01$

TTD vs Gyro resonant interaction

For shear Alfven waves only interactions with $n \neq 0$ are possible. These are referred to as *gyroresonances* because inserting the dispersion relation (129) in the resonance condition (131) yields for the resonance parallel wavenumber

$$k_{\parallel,A} = \frac{n\Omega_c}{\pm V_A - v\mu}, \tag{132}$$

which apart from very small values of $|\mu| \leq V_A/v$ typically equals the inverse of the cosmic ray particle's gyroradius, $k_{\parallel,A} \simeq n/R_L$ and higher harmonics.

In contrast, for fast magnetosonic waves the $n = 0$ resonance is possible for oblique propagation due its compressive magnetic field component. The $n = 0$ interactions are referred to as *transit-time damping*, hereafter TTD. Inserting the dispersion relation (130) into the resonance condition (131) in the case $n = 0$ yields

$$v\mu = \pm V_A/\cos\theta \tag{133}$$

as necessary condition which is independent from the wavenumber value k. Apparently all super-Alfvenic ($v \geq V_A$) cosmic ray particles are subject to TTD provided their parallel velocity $v\mu$ equals at least the wave speeds $\pm V_A$. Hence, equation (133) is equivalent to the two conditions

$$|\mu| \geq V_A/v, \quad v \geq V_A. \tag{134}$$

Additionally, fast mode waves also allow gyroresonances ($n \neq 0$) at wavenumbers

$$k_F = \frac{n\Omega_c}{\pm V_A - v\mu\cos\theta}, \tag{135}$$

Implications

(1) With TTD-interactions alone, it would not be possible to scatter particles with $|\mu| \leq V_A/v$, i.e., particles with pitch angles near 90^o. Obviously, these particles have basically no parallel velocity and cannot catch up with fast mode waves that propagate with the small but finite speeds $\pm V_A$. In particular this implies that with TTD alone it is not possible to establish an isotropic cosmic ray distribution function. We always need gyroresonances to provide the crucial finite scattering at small values of μ.

(2) Conditions (133) and (134) reveal that TTD is no gyroradius effect. It involves fast mode waves at all wavenumbers provided the cosmic ray particles are super-Alfvenic and have large enough values of μ as required by (134). Because gyroresonances occur at single resonant wavenumbers only, see (132) and (135), their contribution to the value of the Fokker–Planck coefficients in the interval $|\mu| \geq V_A/v$ is much smaller than the contribution from TTD. Therefore for comparable intensities of fast mode and shear Alfven waves, TTD will provide the overwhelming contribution to all Fokker–Planck coefficients $D_{\mu\mu}$, $D_{\mu p}$ and D_{pp} in the interval $|\mu| \geq V_A/v$. At small values of $|\mu| < V_A/v$ only gyroresonances contribute to the values of the Fokker–Planck coefficients involving according to (132) and (135) wavenumbers at $k_{\parallel,A} = k_R \simeq \pm n\Omega_c/V_A$.

$$\kappa_{ZZ} = \frac{v\lambda_\parallel}{3}, \quad \lambda_\parallel \simeq \frac{3v}{8}\int_{ep}^{\epsilon} d\mu \frac{(1-\mu^2)^2}{D_{\mu\mu}(\mu)} = \frac{3v\epsilon}{4D_{\mu\mu}(0)} = \frac{3V_A}{4D_{\mu\mu}(0)}$$

Hillas Limit

$$\lambda_{\|}(R_L > L_{\|}) = \infty. \quad (12)$$

Thus, ultrahigh energetic particles with a Larmor radius larger than the scale $L_{\|}$ cannot be confined to the Galaxy. This limit

$$R_{L,H} = \frac{p_H c}{|q| B_0} = L_{\|} \quad (13)$$

is known as the Hillas limit (Hillas 1984). For relativistic particles ($E = cp$) we have for the characteristic energy until which particles can be confined to the Galaxy

$$E_H = |q| B_0 L_{\|}. \quad (14)$$

For protons and by assuming $B_0 = 0.6\text{nT}$ for the galactic magnetic field we have $|q| B_0 = 0.18\text{eV/m} = 5.4 \cdot 10^{15}\text{eV/pc}$. By using that the largest scale of turbulence is $10 - 100\text{pc}$ (see Beck 2007), Eq. (14) becomes

$$E_H = 5.4 \cdot 10^{16}\text{eV} - 5.4 \cdot 10^{17}\text{eV}. \quad (15)$$

Dispersion relation for fast magnetosonic waves read as:

$$\omega_R \simeq ikV_A \quad (5)$$

describing forward ($i = 1$) and backward ($i = -1$) moving fast mode waves. Resonant wavenumber in the gyroresonance case for an isotropic turbulence reads as:

$$k_i^g(\mu) = \frac{n\Omega}{v(\mu\eta - i\epsilon)} = \frac{n}{R_L(\mu\eta - i\epsilon)} \quad (6)$$

where $R_L = v/\Omega$ is gyroradius. Resonant wavenumber in the slab turbulence is

$$k_i^g(\mu) = \frac{n}{R_L\mu} \quad (7)$$

and it is often approximated as

$$k_i^g(\mu) \simeq \frac{n}{R_L}, \quad (8)$$

which actually is undefined at $\mu = 0$ (see Eq. (7)).

At $\mu = 0$ resonant wavenumber for isotropic turbulence (6) will be as follows:

$$k_i^g(\mu = 0) = \frac{n}{R_L\epsilon}, \quad (9)$$

that has no singularity at $\mu = 0$, it is independent of the magnetic field strength ($k_i^g(\mu = 0) = \omega_{p,i}/\gamma c = 1.88 \times 10^{-6}\sqrt{n_e}/\gamma$) and it is by factor of $\epsilon^{-1} = c/V_A$ larger than the Hillas limit [2].

Damping influence

The inclusion of resonance broadening due to wave damping in the resonance function (11) guarantees that this dominance also holds for cosmic ray particles at small pitch angle cosines $\mu \leq |V_a/v|$, unlike the case of negligible wave damping discussed by Schlickeiser and Miller [28]. Therefore, in the

The damping of fast mode waves is caused both by collisionless Landau damping and collisional viscous damping, Joule damping and ion-neutral friction. According to Spanier and Schlickeiser [30] the dominating contribution is provided by viscous damping. Following the analysis and steps performed by Vukcevic [31] resonance function reads as:

$$\mathcal{R}^T_{j\ F}(\mu) = \frac{2.9 \cdot 10^5 \beta V_A^2 \sin^2\theta}{(2.9 \cdot 10^5 \beta V_A^2 k \sin^2\theta)^2 + [v\mu\cos\theta + jV_A]^2}, \qquad (12)$$

where T denotes assumption n=0 in Eq. (11), while for $\mu = 0$ resonance function is

$$\mathcal{R}^T_{j\ F}(0) = \frac{\alpha(1-\eta^2)}{(\alpha(1-\eta^2)k)^2 + V_A^2}, \qquad (13)$$

with $\alpha = 2.9 \cdot 10^5 \beta V_A^2$, $\beta = c_s^2/V_A^2$ and $cos\theta = \eta$.

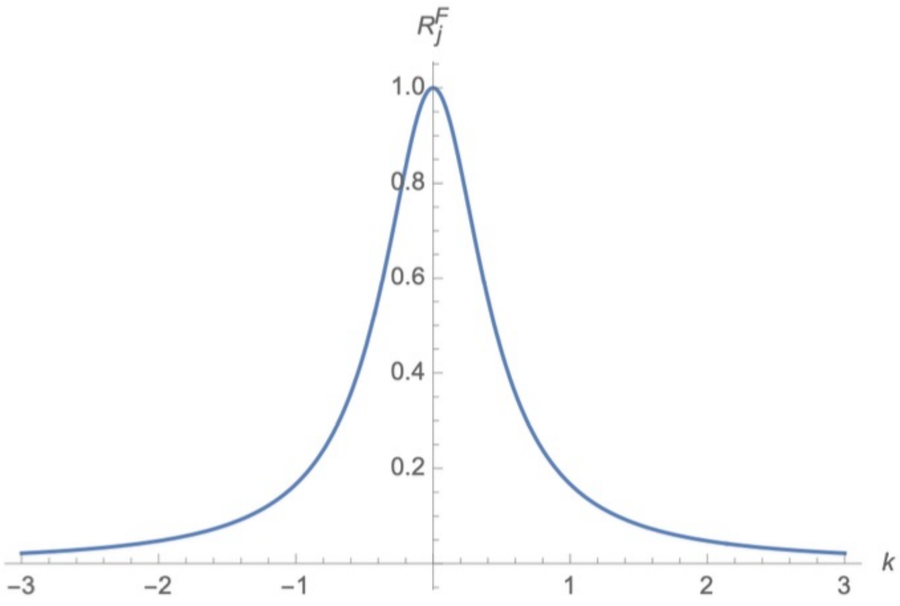


Figure 1: The resonance function given by equation (13) for typical values in the ISM.

The resonance function given by equation (13) is presented in Fig. 1 for typical ISM values for β and V_A. Comparing our result with the result obtained using second order QLT [32] it can be seen that width of the resonance function for damped plasma wave turbulence is independent of **turbulence strength** $\delta B/B_0$. Also, in the case of plasma wave turbulence, which is more realistic than magnetostatic one, it is not correct to simply separate velocity and magnetic field contribution since they are not independent. In the plasma wave turbulence the problem is solved self-consistently and time integration is not independent of turbulence geometry; once isotropic tensor (10) is involved it is necessary to consider relevant plasma modes together with the dispersion relation (5).

Mean free path

$$D^F_{\mu\mu}(\mu=0) \simeq \frac{(q-1)}{\alpha}(k_{min}R_L)^{q-1}(\frac{\delta B}{B_0})^2 \int_{k_{min}R_L}^{\infty} dss^{-q}$$
$$\int_0^1 d\eta\,(1-\eta^4)J_1^2(s\sqrt{1-\eta^2})\frac{1}{(1-\eta^2)^2s^2+\frac{V_A^2R_L^2}{\alpha^2}}, (14)$$

where $s = kR_L$. Note that relevant is only the case $\mu < \epsilon$ and since $\mu \to 0$ integration with respect to η goes from 0 up to 1. Also, in the case of UHECR energy limit will be different from the case of positrons treated in [31] due to different mass of protons end positrons.

$$\lambda^{0F} = \frac{3\kappa}{v} = \frac{3v}{4}\frac{1}{D_{\mu\mu}(\mu=0)}\int_0^{\epsilon} d\mu \;= \frac{3}{4}\frac{V_A}{D^F_{\mu\mu}(\mu=0)} = \frac{3V_A}{4}\frac{\alpha}{(q-1)}(k_{min}R_L)^{1-q}(\frac{B_0}{\delta B})^2\frac{1}{D}, \quad (15)$$

where it is reasonable to take the boundaries in μ integration from 0 to ϵ instead of 0 to 1 since $\mu << \epsilon$, and

$$D = \int_{k_{min}R_L}^{\infty} dss^{-q}\int_0^1 d\eta\,(1-\eta^4)J_1^2(s\sqrt{1-\eta^2})\frac{1}{(1-\eta^2)^2s^2+\frac{V_A^2R_L^2}{\alpha^2}}. \quad (16)$$

Now, we consider two limits: $k_{min}R_L << 1$, and $k_{min}R_L >> 1$, where $k_{min}R_L = E$ and is normalized with respect to E_c, where

$$E_c = \frac{k_c}{k_{min}}Ac^2 = 2\times 10^5 An_e^{1/2}\frac{L_{max}}{10pc}PeV, \quad (17)$$

where A is CR mass, $k_c = \Omega_{0,p}/V_A = \omega_{p,i}/c$, $\Omega_{0,p}$ **is gyrofrequency,** $\omega_{p,i}$ **is plasma frequency** and $k_{min} = 2\pi/L_{max}$. Evaluation of D in these two limits can be found in Vukcevic [31], Appendix A, in details. Note that k_c is different for UHECR from CR treated in the cited paper due to different particle mass.

$k_{min}R_L >> 1$:
In this limit we derive

$$D(E>>1) = \frac{2}{5}\frac{10^{-14}}{q}E^{-(q+2)}, \quad (18)$$

so that mean free path reads as:

$$\lambda^{0F}(E>>1) = \frac{15\alpha}{V_A}\frac{q}{q-1}(\frac{B_0}{\delta B})^2 10^{14}E^3. \quad (19)$$

At relativistic rigidities we find that $\lambda^0 \sim E^3$.

$k_{min}R_L << 1$:
In this case we derive

$$D(E<<1) = \frac{1}{3}\frac{1}{q-1}E^{1-q}, \quad (20)$$

and consequently

$$\lambda^{F0}(E<<1) = \frac{9\alpha}{V_A}(\frac{B_0}{\delta B})^2. \quad (21)$$

In this energy limit the mean free path is constant with respect to E. The mean free path is normalized by factor $\lambda_1 = \frac{9\alpha}{V_A}(\frac{B_0}{\delta B})^2$ which for typical interstellar plasma ($V_A = 30km/s$, $B_0/\delta B \sim 10$ and $q = 5/3$) is $2\times 10^{15}cm$. In Fig. 2 is presented mean free path for damped and undamped case. It is very important to notice that energy limit is not changed compared to undamped case but the value of the mean free path is two orders of magnitude smaller then in the undamped case, which makes damping plasma turbulence very efficient mechanism in confinement of the UHECR. As we have already

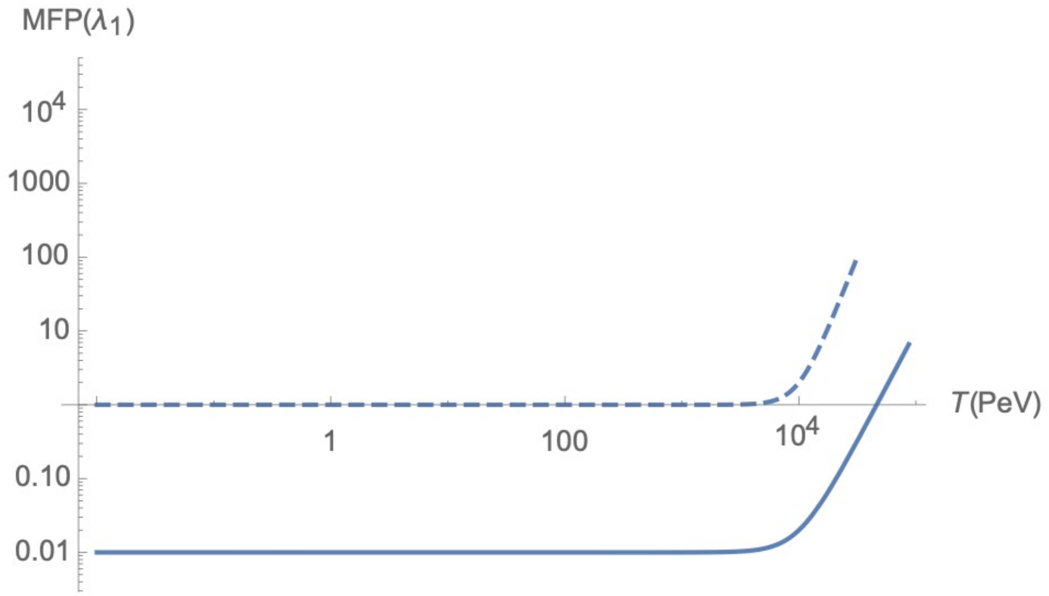


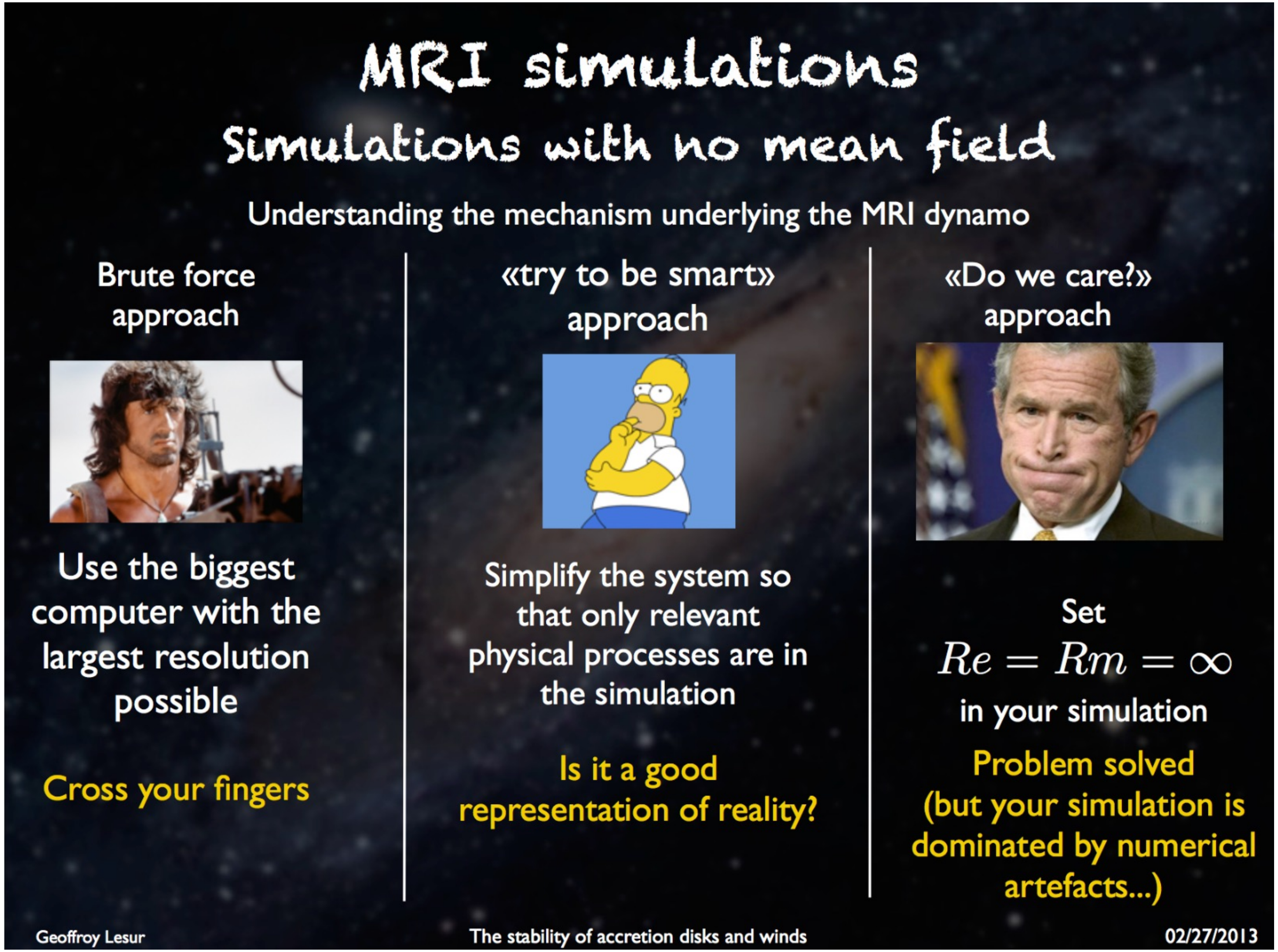
Figure 2: The mean free path for protons versus the particle energy for undamped isotropic wave turbulence (dashed line) and damped case (solid line). Energy at which the mean free path change dependance on energy is $E_c \sim 10^{19}eV$. For slab turbulence at $E_H \sim 10^{15}eV$ the mean free path becomes infinitely large; in the isotropic turbulence that value is enhanced by factor v/V_A due to resonant interaction - gyroresonace for undamped case and TTD resonant interaction in damped case. The second one provides even smaller mean free path due to the broadened resonance function. Mean free path is normalized by λ_1 in pc.

For isotropically distributed interstellar magnetohydrodynamic waves we demonstrated:

- gyroresonance at resonant wave number $k_{res} = (R_L\epsilon)^{-1}$ for $\mu = 0$ is dominant in the case of undamped plasma waves;
- TTD resonant interaction is dominant for $\mu = 0$ in the damped case due to resonance function broadening;
- particle energy limit that can be scattered and therefore confined is enhanced by four orders of magnitude $E_c \sim 10^5 An^{1/2}(L/10pc)PeV$ comparing to Hillas energy;
- mean free path is two orders of magnitude smaller comparing to undamped case.

Below limiting energy the cosmic ray mean free path and the anisotropy exhibit the well known $E^{1/3}$ energy dependence, for $q = 5/3$ denoting the spectral index of the Kolmogorov spectrum for undamped plasma turbulence, while in the damped turbulence both transport parameters remain constant. At energies higher than E_c mean free path and anisotropy steepen to a E^3-dependence. This implies that cosmic rays even close to ultrahigh energies of several tens of EeV can be rapidly pitch-angle scattered by interstellar plasma turbulence, and are thus confined to the Galaxy.

Instead of Summary -- Simulations vs analitical approach



<<smart>>
approach

simplified but
analytical solution,
reliable