Quantum Foundations, Dynamical Born Rule and Intrinsic Triple Interference in Quantum Gravity

Djordje Minic

Virginia Tech

Work done with
N. Bhatta and T. Takeuchi
L. Freidel, J. Kowalski-Glikman and R. Leigh
P. Berglund, T. Hübsch, D. Mattingly and A. Geraci
C. Tze, S. Hur, V. Jejjala, M. Kavic, D. Edmonds, J. Erlich

Outline

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Quantum Gravity (QG)=Gravitized Quantum Theory (GQ)

Summary: Spacetime physics is classical, non-probabilistic, but background independent (generally covariant). Quantum physics is fundamentally probabilistic, but with fixed Born rule. I will argue that **quantum gravity (QG) is"gravitized quantum theory (GQ)" with a dynamical Born rule and intrinsic triple and higher order quantum interference**. (Experimental signature!) I will give formulae for the cosmological constant, Higgs mass, and masses and mixing matrices of quarks and leptons. **Predictions for neutrino masses**. Example of QG=GQ: **metastrings (and metaparticles)** living in **quantum (modular) spacetime**. Dual particles - (fuzzy) dark matter. (Time-varying) dark energy - curvature of dual spacetime. **Dual spacetime - new view on the measurement problem and quantum vs classical!**

Quantum Gravity (QG)=Gravitized Quantum Theory (GQ)

Use the cosmological constant problem to argue for “quantum gravity = gravitized quantum theory” (**QG=QG**) and **emphasize the need for empirical probes** such as **triple and higher-order quantum interference**, etc, at low energies. This talk is based on: [arXiv:2003.00318 [hep-th]], [arXiv:2202.06890 [hep-th]], [arXiv:2203.17137 [gr-qc]], [arXiv:2303.15645 [gr-qc]], [arXiv:2212.00901 [hep-th]], [arXiv:2212.06086 [hep-th]], [arXiv:2307.16712 [hep-th]], [arXiv:2407.06207 [hep-th]], [arXiv:2501.19269 [gr-qc]], [arXiv:2503.20854 [hep-th]].
Message for quantum foundations: Quantum theory emerges from “gravitized quantum theory” (quantum gravity). Quantum measurement and the quantum-classical transition are tied to the implicit averaging over the dual spacetime coordinates.

Geometry of Quantum Theory and Beyond

"Bottom-up" reason for "gravitization of quantum theory". (Minic, Tze) **Geometry of quantum theory** (review by Ashtekar and Schilling) - maximally symmetric geometry of CP^N, with Fubini-Study (FS) metric (maximally symmetric Fisher metric of information theory (Wootters)) $ds_F^2 = \sum_i \frac{dp_i^2}{p_i}, \sum_i p_i = 1$ where p_i are probabilities. Let $p_i \equiv x_i^2$ so that we have a sphere in a Euclidean space $ds_F^2 = \sum_i dx_i^2, \sum_i x_i^2 = 1$. If $i = 1, ..., 2(N+1)$, (x_is real and imaginary components of the wavefunction), then we have an odd-dimensional sphere S^{2N+1}, which is always a $U(1)$ bundle of CP^N (with FS metric). **Compatibility of Fisher with the symplectic form - equations of motion - gives the complex structure, that is i in the Schrödinger equation.**

Schrödinger equation is the geodesic equation on CP^n.
Born rule - the FS distance on CP^n: $ds^2_{FS}(1,2) = 4(1 - |\langle\psi_1|\psi_2\rangle|^2)$
Note $CP^n = U(n+1)/U(n) \times U(1)$; $U(n+1)$ - unitary evolution; $U(n)$ - non-Abelian Berry phase; $U(1)$ - the complex phase of the wave function; Entanglement - Segre embedding of the product of lower dimensional CP^ns into a higher dimensional CP^N. (*Density matrices - Bures metric - analog of FS/Fisher*). From the Schrödinger equation derive (Aharonov and Anandan)

$$2\hbar ds_{FS} = \Delta E dt, \qquad (1)$$

where ΔE is the dispersion of energy, and ds_{FS} is the Fubini-Study metric on CP^n (which is directly related to the Fisher metric).
Quantum spacetimes (topology change) - no unique timelike Killing vector - and thus ΔE is state dependent, so the geometry is state dependent, and therefore, dynamical. (Minic, Tze).
Thus: Generally covariant quantum theory with a dynamical Born rule.

Most important: **Gravity violates assumptions of Chentsov theorem about the uniqueness of the Fisher metric from sufficient statistics (BGHMM, '25)**

Chentsov theorem: *Fisher metric is the unique information metric under sufficient statistics and identical independent measurements*

(**1**) Data is a set of permanent results from independent identically distributed (i.i.d.) measurements - in quantum gravity the data is not i.i.d. but instead non-Markovian, as each successive recording of a data point affects the next measurement (due to equivalence principle; to store data requires energy which is a gravitational charge and thus back-reacts). and (**2**) sufficient statistics cannot be generated and passed between observers in quantum gravity due to different boundary conditions (gravity cannot be screened). **As a result, the question of whether the Born rule is fixed or dynamical in quantum gravity becomes an empirical question.**

Geometry of Quantum Theory and Beyond

What would be the first experimental consequence of “dynamical Born rule” or “gravitized quantum theory”? (Beglund, Geraci, Hübsch, Mattingly, Minic)

Triple and higher-order quantum interference! (Experiment possible in the next few years.)

Note - the canonical quantum theory does not have intrinsic triple quantum interference (consequence of the Born rule and the fixed geometry of the complex projective space). **In quantum theory: triple interference is a measurement of zero.**

Current experimental bounds (photonic) - rather weak (10^{-3}).

Neutrino bounds expected to be surprisingly similar (and to be measured at JUNO).(Huber, Minakata, Minic, Pestes, Takeuchi).

No tests of the Born rule with gravity!

Geometry of Quantum Theory and Beyond

In more detail (Sorkin): Classically, we have addition of probabilities

$$P_n(A, B, C, \cdots) = P_1(A) + P_1(B) + P_1(C) + \cdots , \qquad (2)$$

for any number of paths. Quantum mechanically, we have for two paths $P_2(A, B) = |\psi_A + \psi_B|^2$ or more explicitly

$$|\psi_A|^2 + |\psi_B|^2 + (\psi_A^* \psi_B + \psi_B^* \psi_A) \equiv P_1(A) + P_1(B) + I_2(A, B) \quad (3)$$

where the last term

$$I_2(A, B) = P_2(A, B) - P_1(A) - P_1(B) \qquad (4)$$

is the "interference" of the two paths A and B. **Non-vanishing double-path interference, $I_2(A, B) \neq 0$, distinguishes quantum theory from the classical one.**

Geometry of Quantum Theory and Beyond

The **Born rule** dictates that all the superimposed paths only interfere with each other in a pairwise manner. For instance, for three paths we have $P_3(A,B,C) = |\psi_A + \psi_B + \psi_C|^2$

$$P_2(A,B) + P_2(B,C) + P_2(C,A) - P_1(A) - P_1(B) - P_1(C), \qquad (5)$$

where only pairwise interferences between the pairs (A,B), (B,C), and (C,A) appear.

It is clear from the above that in order for there to be a non-linear correction in an interference pattern the Born rule must be relaxed.

Consider **a triple slit experiment**: Since only pairwise interferences between the pairs (A, B), (B, C), and (C, A) appear, it makes sense to define any deviation from this relation as the intrinsic triple-path interference $I_3(A, B, C)$ (Sorkin)

$$P_3(A,B,C)-P_2(A,B)-P_2(B,C)-P_2(C,A)+P_1(A)+P_1(B)+P_1(C). \tag{6}$$

(This can be easily generalized for the case of n-paths.) For both classical and quantum theory, this intrinsic triple-path interference is zero for any triplet of paths. **Experimental confirmation of $I_3 = 0$ would be a confirmation of the Born rule.**
Weak bounds were placed on the parameter ($\kappa \sim 10^{-3}$) in photonic experiments (**No check of the Born rule in gravity!**)

$$\kappa = \frac{\varepsilon}{\delta}, \quad \varepsilon = I_3(A,B,C), \quad \delta = |I_2(A,B)| + |I_2(B,C)| + |I_2(C,A)|. \tag{7}$$

The claim (Berglund, Geraci, Hubsch, Mattingly, Minic) is that **with quantum gravitational degrees of freedom turned on, one can get $I_3 \neq 0$,** but for that one needs gravitized quantum theory, with **observer dependent spacetime (Hilbert spaces) and dynamical Born rule**. The generalized probability in this approach to quantum gravity is given by (analogy with non-linear optics), which, geometrically, looks very Finsler-like

$$P = g_{ab}(\psi)\,\psi_a\psi_b \equiv \delta_{ab}\,\psi_a\psi_b + \gamma_{abc}\,\psi_a\psi_b\psi_c + \dots, \qquad (8)$$

where a, b, c are state-space indices and with (schematically) - Schrödinger plus Nambu quantum theory (Minic, Tze)

$$\frac{\mathrm{d}\psi_a}{\mathrm{d}\tau} = E_{ab}\psi_b + \Gamma_{abc}\,\psi_b\psi_c, \qquad (9)$$

where τ is the appropriate evolution parameter (and higher order generalizations). (Schrödinger's evolution - geodesic eq on CP^N, for a fixed Hilbert space.)

Comments: Nambu mechanics - based on volume preserving diffeos. Nambu bracket - generalization of the Poisson bracket. (for example: $\{f, g, h\} \equiv \epsilon_{ijk}\partial_i f \partial_j g \partial_k g$)
Example: asymmetric top can be written as Nambu mechanics. $dO/dt \equiv \{H_1, H_2, O\}$ where $h_1 = aL_1^2 + bL_2^2 + cL_3^2 \equiv E$ and $h_2 = L_1^2 + L_2^2 + L_3^2 \equiv L^2$.
Nambu quantum theory - Schrödinger representation. (Just replace L_1, L_2, L_3 by ψ_1, ψ_2, ψ_3.)
Matrix representation - cubic and higher order matrices. (General probability theories - GPT.)
Perturb the canonical Schrödinger evolution by a Nambu-like quantum theory (knows about ψ_3, modeling "quantum spacetime") in order to model intrinsic triple interference.

Gravitized quantum theory: the complex projective spaces generalized (for example: Bloch sphere could become Riemann surface of infinite genus) Higher order correlations - dynamical Born rule. Handles - higher components ψ_3 etc. Classical limit - topological branching (the metric is degenerate - zero). Re-summation of the infinite number of multilinear extensions - euclidean GR like (Finsler) theory in the general space of states (that includes ψ_3 etc). Canonical quantum theory - maximally symmetric limit (ex: average over the infinite number of handles). **Non-linear optics analog: Poincare sphere of polarization in linear optics (direct analog of the Bloch sphere in quantum theory) gets deformed in non-linear optics.**

Geometry of Quantum Theory and Beyond

Note: Effective triple interference - possible in non-linear optical media! (Instead of ψ, non-linear waves; instead of probability P - non-linear/cubic energy density.) **"Smoking gun": Talbot effect on a diffraction grating $\rightarrow$ non-linear Talbot effect.** *Effect of decoherence different from the observable signatures of the non-linear Talbot effect.* **Thus intrinsic triple interference with quantum gravity degrees of freedom - analogous to a non-linear "quantum spacetime medium".** (*Note: non-linear quantum theory with fixed Hilbert spaces is NOT GQ!*) **Also: Need to go beyond traditional GPT - non-Markovian nature of gravity due to its memory!** Computation of the vacuum energy/cosmological constant (below), suggests a low energy scales: $10^{-3}eV$ or $10^{-4}m$.

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The CC Problem in QFT

Cosmological constant Λ (a parameter in Einstein's eqs $G_{\mu\nu} + \Lambda_{cc} g_{\mu\nu} = 8\pi G T_{\mu\nu}$) has been measured (supernovae, CMB, large scale structure). It corresponds to the (quantum) vacuum energy $\Lambda_{cc}/(8\pi G) \sim (10^{-3} eV)^4$. The natural Planckian value is 10^{124} times off $(10^{19} GeV)^4$ - **the cosmological constant problem**. (*Analogous to the stability of atoms.*) Need to explain the observed value and its radiative stability.
Let us start with the QFT vacuum partition function (free scalar):

$$Z_{vac} = \int D\phi e^{-\int \frac{1}{2}\phi(-\partial^2+m^2)\phi} = \sqrt{\frac{\#}{\det(-\partial^2+m^2)}} \tag{10}$$

which we can rewrite as

$$Z_{vac} = e^{-\frac{1}{2}\mathrm{Tr}\log(-\partial^2+m^2)} \tag{11}$$

The CC Problem in QFT

In momentum space, $-\partial^2 = k^2$, and also

$$-\frac{1}{2}\log(k^2+m^2) = \int \frac{\mathrm{d}l}{2l} e^{-(k^2+m^2)l/2} \tag{12}$$

where the Schwinger parameter l is a worldline parameter associated with a **particle (quantum of the field ϕ)**.

The CC Problem in QFT

Note that after taking the trace we have

$$\int \frac{\mathrm{d}^D k}{(2\pi)^D} \log(k^2 + m^2) = \int \frac{\mathrm{d}^{D-1} k}{(2\pi)^{D-1}} \frac{\omega_k}{2} \tag{13}$$

because

$$\int \frac{\mathrm{d}l}{2l} \int \frac{\mathrm{d}k^0}{2\pi} e^{-(k^2+m^2)l/2} = \frac{\omega_k}{2} \tag{14}$$

where $\omega_k^2 = k^2 + m^2$ with ω_k equivalent to k_0 on-shell.

The CC Problem in QFT

Thus, vacuum energy density in D spacetime dimensions becomes

$$\rho_0 = \int \frac{\mathrm{d}^{D-1}k}{(2\pi)^{D-1}} \frac{\omega_k}{2} \sim \Lambda_D \qquad (15)$$

Λ_D is the volume of energy-momentum space. $(\sum_k \frac{1}{2}\hbar\omega_k,\ \hbar = 1)$
This is a divergent expression (to be regularized) that leads to **the cosmological constant problem**. (*Weinberg's classic review.*)
The cosmological constant in 4d is (Einstein's equations $G_{ab} + \Lambda_{cc} g_{ab} = 8\pi G_N T_{ab}$) so that $\Lambda_{cc} \sim \rho_0 G_N \sim \rho_0 l_P^2$.

The CC Problem in QFT

Note that the **vacuum partition function** is also

$$Z_{vac} = \langle 0 | e^{-iH\tau} | 0 \rangle = e^{-i\rho_0 V_D} \tag{16}$$

where V_D is the volume of D-dimensional spacetime, and ρ_o is the vacuum energy density.
Furthermore $Z_{vac} = exp(Z_{S^1})$ where Z_{S^1} is the partition function on S^1 in the world-line formulation

$$Z_{s^1} = V_D \int \frac{\mathrm{d}^D k}{(2\pi)^D} \int \frac{\mathrm{d}l}{2l} e^{-(k^2+m^2)\frac{l}{2}} \tag{17}$$

Thus the **vacuum energy density** is given by (scaling as before)

$$\rho_0 = \frac{iZ_{S^1}}{V_d} \sim \Lambda_D \tag{18}$$

The CC Problem in QG/String Theory

For the case of a **bosonic string**, instead of one particle we have an infinite tower of particles with mass spectrum (Polchinski '86; Polchinski's String theory book) - graviton $(h = 1, \bar{h} = 1)$

$$m^2 = \frac{2}{\alpha'}(h + \bar{h} - 2) \tag{19}$$

Thus, summing over the physical string states

$$\sum_{\text{p.s}} Z_{S^1} = \sum_{h,\bar{h}} V_D \int \frac{\mathrm{d}l(2\pi l)^{-D/2}}{2l} \int \frac{\mathrm{d}\theta}{2\pi} e^{i(h-\bar{h})\theta} e^{-\frac{2}{\alpha'}(h+\bar{h}-2)\frac{l}{2}} \tag{20}$$

where we have imposed the level matching $h = \bar{h}$ (or $\delta_{h,\bar{h}}$).

The CC Problem in QG/String Theory

Define $\tau = \theta + i\frac{l}{\alpha'} \equiv \tau_1 + i\tau_2$.
We get the partition function of a bosonic string on T^2

$$Z_{T^2} = V_D \int \frac{\mathrm{d}\tau \mathrm{d}\bar{\tau}}{2\tau_2} (4\pi^2\alpha'\tau_2)^{-D/2} \sum_h q^{h-1} \bar{q}^{\bar{h}-1} \tag{21}$$

where $q \equiv e^{2\pi i\tau}$. This can be derived directly from the Polyakov path integral.

The CC Problem in QG/String Theory

Note that we can rewrite, with $l \equiv \alpha' \tau_2$,

$$(4\pi^2\alpha'\tau_2)^{-D/2} = \int \frac{\mathrm{d}^D k}{(2\pi)^D} e^{-k^2 \frac{l}{2}} \tag{22}$$

Thus, as in QFT we can write $Z_{T^2} \equiv V_D \int \frac{\mathrm{d}^D k}{(2\pi)^D} f(k^2) \sim V_D \Lambda_D$ with

$$\Lambda_D \equiv \int \frac{\mathrm{d}^D k}{(2\pi)^D}; \quad f(k^2) \equiv \int_F \frac{\mathrm{d}^2\tau}{2\tau_2} e^{-k^2\alpha'\tau_2/2} \sum_h q^{h-1} \bar{q}^{h-1} \tag{23}$$

where F is the fundamental domain. Note that $f(k^2)$ is dimensionless, so it does not contribute to the scaling of Z_{T^2} and the vacuum energy $\rho_0 \sim Z_{T^2}/V_D$.

The CC Problem in QG/String Theory

The only difference is that in QFT the region of integration is

$$|\tau_1| < \frac{1}{2}, \quad \tau_2 > 0. \tag{24}$$

In string theory, because of modular invariance,

$$|\tau_1| < \frac{1}{2}, \quad |\tau| > 1. \tag{25}$$

So, the cosmological constant is UV finite(!) in string theory, but still $\rho_0 \sim Z_{T^2}/V_D \sim \Lambda_D$ (!) - **the CC problem persists in string theory!**

Unbroken SUSY - flat space or AdS. Broken SUSY - still $\rho_0 \sim \Lambda_D$.

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Quantum Gravity, Phase Space and Observation

Following FKLM [2212.00901], [2303.17495] (see also BHM, [2212.06086]) we return to Z_{S^1}, setting $m = 0$ for convenience and with p denoting the momentum,

$$Z_{S^1} = V_D \int \frac{\mathrm{d}\tau}{2\tau} \int \frac{\mathrm{d}^D p}{(2\pi)^D} e^{-\frac{p^2 \tau}{2}} \tag{26}$$

But the spacetime volume is given by $V_D = \int \mathrm{d}^D q$. We therefore consider the **phase space** expression

$$Z_{S^1} = \int \frac{\mathrm{d}\tau}{2\tau} Z(\tau), \quad Z(\tau) = \int \frac{\mathrm{d}^D q}{(2\pi)^D} \int \mathrm{d}^D p e^{-\frac{p^2 \tau}{2}} \equiv \mathrm{Tr} e^{-\frac{p^2 \tau}{2}} \tag{27}$$

where Tr is in *phase space*.

In $D = 4$ we then get

$$Z(\tau) = \prod_{i=1}^{4} \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}q_i \int_{-\infty}^{\infty} \mathrm{d}p_i e^{-\frac{p_i^2 \tau}{2}} \tag{28}$$

or by discretizing phase space

$$Z(\tau) = \left(\frac{\lambda\epsilon}{2\pi} \sum_{k,\tilde{k}\in\mathbf{Z}} \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}\tilde{x} e^{-\frac{(\tilde{x}+k)^2 \epsilon^2 \tau}{2}} \right)^4 \tag{29}$$

where $p \to \epsilon\tilde{x}, q \to \lambda x$ with $\lambda\epsilon = \hbar$.

This is divergent but restrict the sum to finite range, **modular regularization** (Following papers on modular polarization in quantum theory and modular spacetime by FLM, '15, '16 - see slides in what follows)

$$Z(\tau) = \left(\frac{\lambda \epsilon}{2\pi} \sum_{k=0}^{N_q - 1} \sum_{\tilde{k}=0}^{N_p - 1} \int_0^1 \mathrm{d}x \mathrm{d}\tilde{x} e^{-\frac{(k+\tilde{x})^2 \epsilon^2 \tau}{2}} \right)^4 \quad (30)$$

where N_q, N_p count the number of unit cells in the spacetime and momentum space dimensions, respectively.
Now, define

$$l \equiv N_q \lambda, \quad \text{and} \quad \Lambda \equiv N_p \epsilon \quad (31)$$

where then $l^4 \equiv V_4$ is the size (volume) of spacetime and Λ^4 is the size (volume) of energy-momentum space, and $N = (N_p N_q)^4 \in \mathbf{Z}$.

Thus,

$$l^4\Lambda^4 = N, \quad \text{or} \quad \Lambda^4 = \frac{N}{l^4} \tag{32}$$

But there is actually an **upper bound on** $\rho_0 \sim \Lambda^4 \leq \frac{N}{l^4}$ in $D = 4$ due to $exp(-p^2\tau/2) \leq 1$. **Note: N should be thought of as entropy. One can be in a phase cell or not! Thus there are** 2^N **possibilities, and its logarithm scales as N.**
The same bound also holds in string theory following our earlier calculation of the partition function of the bosonic string on T^2 in $D = 4$. (*We'll show that the same bound holds in QFT and cosmological phase transitions described by an effective potential.)*

$$\rho_0 \leq \frac{N}{l^4} \tag{33}$$

Quantum Gravity, Phase Space and Observation

We now consider the **Bekenstein bound** in a space with a cosmological horizon, ie assuming that the cosmological constant is positive and we have a dS spacetime. (*This is a feature of semiclassical gravity, and also of gravitational thermodynamics.*) In static coordinates, dS spacetime metric is

$$\mathrm{d}s_{dS}^2 = -(1 - \frac{r^2}{r_{CH}^2})dt^2 + \frac{dr^2}{(1 - \frac{r^2}{r_{CH}^2})} + r^2 \mathrm{d}\omega_{S^2}^2 \qquad (34)$$

where $l \equiv r_{CH}$, the cosmological horizon, is the size of the observed spacetime.

By identifying the above quantum number N with the gravitational entropy, the **Bekenstein bound ($S_{grav} = l_P^{-2} Area$)** becomes

$$N \leq \frac{l^2}{l_P^2} \tag{35}$$

(*Experimental consequence for black holes - gravitational wave "echoes" - in the quantum chaos phase, because N is large - quantum scars?*)

Combine the Bekenstein bound with the bound on ρ_0 ($\rho_0 \leq N/l^4$)

$$\rho_0 \leq \frac{1}{l^2 l_P^2} \tag{36}$$

Mixing of the UV (l_P) and the IR (l) scales.

With the cosmological constant in $D = 4$ dimensions, $\Lambda_{c.c} = \rho_0 l_P^2$ we then get the bound (model independent!)

$$\Lambda_{c.c} \leq \frac{1}{l^2} \tag{37}$$

Thus, the natural energy scale, $\epsilon_{c.c}$ associated with the vacuum energy density (and model independent) is

$$\rho_0 = \epsilon_{c.c}^4 \sim \frac{1}{l^2 l_p^2} \tag{38}$$

(the corresponding natural length scale, $l_{c.c} \simeq 1/\epsilon_{c.c}$), and we get the **see-saw formula** (FKLM) - also in full QFT (cosmological phase transitions etc) [probe - gravitational interferometry]

$$l_{c.c} \simeq \sqrt{l\, l_p} \tag{39}$$

Quantum Gravity, Phase Space and Observation

Note (integration over the Schwinger parameter can be absorbed in the renormalization of the Newton constant). Also:

- with $l \sim 10^{28} m$ we get $l_{c.c} \simeq 10^{-4}$ m or 10^{-3} eV in agreement with observations. Use for other predictions.
- natural with $\rho_0 \to 0$ when $l \to \infty$, and l is the IR scale
- radiativelly stable since no UV dependence
- the CC is small because the universe is filled with stuff (large number of degrees of freedom (dof) $N \sim 10^{124}$)
- N is large because fluctuations scale as $\frac{1}{\sqrt{N}}$ - stability of the universe (Schrödinger's argument "Why are atoms small?")
- N_i (where i is t, x, y, z) is $N^{1/4} \sim 10^{31}$ (Avogadro number for spacetime atoms: QG Brownian motion - Zurek et al.).

Note further the **contextuality** of the argument: the measurement of a quantum observable depends on which commuting set of observable are within the same measurement set of observable, ie quantum measurements depend on the *context*!

- First, ϵ is NOT a cut-off, as ϵ and λ can be arbitrary, though have to satisfy $\lambda\epsilon = \hbar$
- Second, ϵ^4 is replaced by N, which is the new quantum number, and the size of spacetime, $l = r_{CH}$
- N is determined by the Bekenstein bound–N is related to l and l_P, which is where gravity enters via $G_N \sim l_P^2$

Effective field theory (EFT) does not "see" phase space and N (EFT lives in classical spacetime, OR momentum space) and it does not mix UV and IR. (However, see the work of Reuter et al.)

Note on EFT: the QFT partition function is defined as $Z(J) \equiv e^{iW(J)} = \int D\phi \; e^{i[S(\phi)+J\phi]}$, where $W(J)$ is the generating functional of vacuum correlation functions, and it represents a direct analogue of the partition function for a particle on a circle, or a string on a torus!

Given $W(J)$, we can define its Legendre transform to obtain $\Gamma(\phi)$, the effective action, as $\Gamma(\phi) \equiv W(J) - \int \mathrm{d}^4x \; J(x)\phi(x)$. The leading term in the expansion of $\Gamma(\phi)$ is the effective potential, $\Gamma(\phi) \equiv \int \mathrm{d}^4x[-V_{\text{eff}}(\phi) + \dots]$, the minimum of which defines the vacuum energy in QFT.

Then by introducing the Schwinger parametrization we can obtain that $\Gamma(\phi) \sim \int \mathrm{d}^4x \int \frac{\mathrm{d}^4k}{(2\pi)^4} \int \frac{\mathrm{d}r}{r} \; e^{-U(k^2,\phi)\,r/2}$, where the exponent in the above integral is bounded by 1. (Also at finite temperature!)

By applying modular regularization to the crucial phase space factor together with the Bekenstein bound we get the already derived result for the bound on the vacuum energy!

QG = GQ and Experiment:

Summary - The essence of the vacuum energy calculation: $Z_{vac} = \langle 0| e^{-iH\tau} |0\rangle = e^{-i\rho_0 V_D}$ where V_D is the volume of D-dimensional spacetime, and ρ_o is the vacuum energy density. Particles (p): $Z_{vac} = exp(Z_{S^1}) \equiv exp(Z_p)$; Strings (s): $Z_{vac} = exp(Z_{T^2}) \equiv exp(Z_s)$. So, $\rho_0 \sim Z_{p,s}/V_D \sim \Lambda^D$ ($V_D \sim l^D$) In both cases: $Z_{p,s} \sim V_D \int \frac{\mathrm{d}^D k}{(2\pi)^D} ... \sim l^D \Lambda^D$. *Modular regularization of phase space volume:* $l^D \Lambda^D = N$. Holography $N \sim l^{D-2}/l_P^{D-2}$. In $D = 4$ we get that $\rho_0 \sim 1/(l^2 l_P^2)$ and thus $m_\Lambda \sim \sqrt{M M_P}$. (Here: M - Hubble scale; M_P - Planck scale.) $M = 10^{-34} eV$; $M_P = 10^{19} GeV$ Note, M and M_P are **contextual** *IR* and *UV* scales. **In general: dynamical phase space (because of dynamical spacetime!) and thus, dynamical quantum phase space. Dynamical Born rule. QG=GQ.**

QG = GQ and Experiment:

So what? Extra evidence: Let us repeat the same logic for some *effective action* that leads to the masses of elementary particles. First, the Higgs - $M_H^2 \sim g_s^2 M_s^2$, where the relevant *IR* scale is the **Bjorken-Zeldovich scale (BZ),** M_{BZ}**.** Match spacetime and matter entropies: $N \sim l^2/l_P^2$ **to** l^3/l_{BZ}^3, so that $l_{BZ}^3 \sim l l_P^2$. $(M_{BZ}^3 \sim M M_P^2 \sim (7 MeV)^3)$. The UV scale is M_P. Obtain, $m_H \sim \sqrt{m_\Lambda M_P}$. (Here we use some stringy formulae for m_H.) Repeat for the Standard Model (SM) fermions: $m_f \sim g_s M_s$ and use the phase-space-like structure of modular spacetime. The relevant IR scale - M_{BZ} and the relevant UV scale - the heaviest fermion scale. (Use SM criticality to relate the heaviest fermion masses to the Higgs mass.) Two expressions are possible:

$m_f \sim M_{IR}\sqrt{\frac{M_{UV}}{M_{IR}}} \sim \sqrt{M_{IR} M_{UV}}$ or $m_f \sim M_{IR}\sqrt{\frac{M_{IR}}{M_{UV}}}$

QG = GQ and Experiment - Results 1:

CC: $m_\Lambda \sim \sqrt{M M_P} \sim 10^{-3} eV$; $m_H \sim \sqrt{m_\Lambda M_P} \sim 125 GeV$; m_H and RG: Determine m_t, m_b and m_τ from m_H (SM criticality). $M_{BZ}^3 \sim M M_P^2 \sim (7 MeV)^3$. Then:

$$m_c \sim \sqrt{M_{BZ}\, m_t} = M_{BZ}\sqrt{\frac{m_t}{M_{BZ}}} \sim 1.10\ (1.27)\,\text{GeV}.$$

$$m_s \sim \sqrt{M_{BZ}\, m_b} = M_{BZ}\sqrt{\frac{m_b}{M_{BZ}}} \sim 171\ (93.4)\text{MeV}.$$

$$m_u \sim M_{BZ}^2/m_c \sim M_{BZ}\sqrt{\frac{M_{BZ}}{m_t}} \sim 10^{-2} M_{BZ} \sim 10^{-1}\ (2.16)\text{MeV}.$$

$$m_d \sim M_{BZ}^2/m_s \sim M_{BZ}\sqrt{\frac{M_{BZ}}{m_b}} \sim 10^{-1} M_{BZ} \sim 1\ (4.67)\text{MeV}.$$

$$m_\mu \sim \sqrt{M_{BZ}\, m_\tau} = M_{BZ}\sqrt{\frac{m_\tau}{M_{BZ}}} \sim 112\ (106)\text{MeV}.$$

$m_e \sim \frac{M_{BZ}^2}{m_\mu} \sim M_{BZ}\sqrt{\frac{M_{BZ}}{m_\tau}} \sim 464\ (511)\text{keV}$. **Prediction for:** (**Neutrino masses**) $m_3 \sim m_H^2/M_{SM} \sim (10^{-2} - 10^{-1})\text{eV}$. $m_2 \sim \sqrt{m_\Lambda m_3} \sim (10^{-2.5} - 10^{-2})\text{eV}$. $m_1 \sim \frac{{m_\Lambda}^2}{m_2} \sim (10^{-4})\text{eV}$.

QG = GQ and Experiment - Results 2:

CKM matrix (quark mixing matrix)

$$|V_{cb}| \sim \frac{M_{BZ}}{\sqrt{m_b\, m_d}} \sim \sqrt{\frac{M_{BZ}}{m_b}}\sqrt{\frac{M_{BZ}}{m_d}} \sim 0.050 \quad (0.041), (\rightsquigarrow \theta_{23})$$

$$|V_{td}| \sim \frac{M_{BZ}}{\sqrt{m_b\, m_s}} \sim \sqrt{\frac{M_{BZ}}{m_b}}\sqrt{\frac{M_{BZ}}{m_s}} \sim 0.011 \quad (0.008)(\rightsquigarrow \theta_{12})$$

$$|V_{ub}| \sim \frac{M_{BZ}}{\sqrt{m_b\, m_b}} \sim \sqrt{\frac{M_{BZ}}{m_b}}\sqrt{\frac{M_{BZ}}{m_b}} \sim 0.002 \quad (0.003)(\rightsquigarrow \theta_{13})$$

PMNS (neutrino mixing matrix: $M_{BZ} \to m_\Lambda$)

$$|U_{\mu 3}| \sim \frac{m_\Lambda}{\sqrt{m_3 m_1}} \sim \sqrt{\frac{m_\Lambda}{m_3}}\sqrt{\frac{m_\Lambda}{m_1}} \sim 0.50, \quad (0.63)$$

$$|U_{\tau 1}| \sim \frac{m_\Lambda}{\sqrt{m_3 m_2}} \sim \sqrt{\frac{m_\Lambda}{m_3}}\sqrt{\frac{m_\Lambda}{m_2}} \sim 0.13, \quad (0.26)$$

$$|U_{e3}| \sim \frac{m_\Lambda}{\sqrt{m_3 m_3}} \sim \sqrt{\frac{m_\Lambda}{m_3}}\sqrt{\frac{m_\Lambda}{m_3}} \sim 0.06, \quad (0.14) \text{ (Similar structure!)}$$

Coincidence? 20 parameters written in terms of Hubble and Planck scales (and the Standard Model scale)?

QG=Gravitization of Quantum Theory

Suppose one believes all this. How does one formulate QG=GQ?

Central intuition: **Quantum Relativity** in analogy with **Classical Relativity**

Classical relativity:

a) special relativity - motivated by EM - (*Minkowski spacetime/geometry, relativity of simultaneity*),

b) relativistic field theory (*reps of Lorentz/Poincare, particles/antiparticles*),

c) general relativity (*dynamical classical spacetime*)

Spacetime relativity - first (classical) relativity (both spacetime and matter classical).

QG=Gravitization of Quantum Theory

Quantum relativity: (FLM, '13, '14, '15, '16, '17)
A) QM from quantum spacetime (**modular spacetime, Born geometry, relative locality**),
B) QFT (**metafields/metaparticles**),
C) **gravitized quantum theory** (**dynamical quantum spacetime**, **dynamical Born geometry**, **metastrings**, metaparticles - dark matter, geometry of dual spacetime - dark energy)
QM/QFT - second (quantum) relativity (matter quantum, spacetime classical).
QG=GQ - third (QG) relativity (spacetime/matter quantum).
(Third relativity - Finkelstein; Wheeler) **Dynamical quantum probabilities. Background independent quantum theory.**

QG=Gravitization of Quantum Theory

Modular variables (Aharonov): suppose we define our physics on a lattice (lattice QFT). Continuum via Wilsonian RG (via path integral). Lattice is classical, physics is quantum. In quantum theory we need a **lattice (l) and a dual lattice ($\tilde{l}$)**! (as noticed by Zak)

Instead of considering the standard commutation relations between the position and momentum operators, $[q, p] = i\hbar$, take the generators of translations in **phase space**

$$\hat{U}_a = e^{\frac{i}{\hbar}\hat{p}a}, \quad \hat{V}_{\frac{2\pi\hbar}{a}} = e^{\frac{i}{\hbar}\hat{q}\frac{2\pi\hbar}{a}}, \quad \Longrightarrow [\hat{U}_a, \hat{V}_{\frac{2\pi\hbar}{a}}] = 0 \tag{40}$$

In terms of **modular variables** a la Aharonov et al,

$$[\hat{q}]_a \equiv \hat{q} \bmod a \quad [\hat{p}]_{\frac{2\pi\hbar}{a}} \equiv \hat{p} \bmod \frac{2\pi\hbar}{a} \quad \Longrightarrow [[\hat{q}]_a, [\hat{p}]_{\frac{2\pi\hbar}{a}}] = 0 \tag{41}$$

QG=Gravitization of Quantum Theory

Modular variables are covariant (modular energy, modular time as well).

Take fundamental length λ and energy ϵ, so that $\lambda\epsilon \equiv \hbar$. **Modular variables are non-local (but consistent with causality - origin of the uncertainty principle)**.

Contextuality: *in a double slit experiment the parameters* λ *and* ϵ *are contextual to the experiment*.

QG=Gravitization of Quantum Theory

Explicit non-locality: Take $H = \frac{p^2}{2m} + V(q)$ and write the Heisenberg equation of motion for $e^{ipR/\hbar}$, or equivalently $[p]_R$ (R - contextuality parameter, such as the distance between two slits in the double slit interference experiment). (Aharonov et al)

$$\frac{d[p]_R}{dt} = ... \frac{V(q + R/2) - V(q - R/2)}{R} \tag{42}$$

Quantum mechanics=non-locality (from contextual modular variables) plus causality (compatibility with Lorentz). Now, reformulate quantum mechanics (QM) using (covariant) modular variables via modular spacetime. (**Quantum theory tells us something new about quantum spacetime!**)

QG=Gravitization of Quantum Theory

Introduce **modular spacetime**. First: what is **modular space**? (FLM, '16)

Modular space is the space of all commuting subalgebras of the Heisenberg-Weyl algebra.

Note $[q, p] = i\hbar$ - Heiseinberg-Weyl algebra, whereas $[[q]_a, [p]_{2\pi\hbar/a}] = 0$ - commuting subalgebra of Weyl-Heisenberg.

Mackey's theorem: **The space of all commuting subalgebras of the Heisenberg-Weyl algebra is a self-dual phase space lattice lifted to Heisenberg-Weyl.**

Use covariant modular variables - **modular spacetime** of d spacetime dimensions.

Note (FLM): **phase space - symplectic structure** $Sp(2d)$ - ω_{ab}.
Self-dual lattice (l plus $\tilde{l}$) - **doubly-orthogonal** $O(d,d)$ - η_{ab}.
To **define the vacuum** on this self-dual lattice - **need doubly metric structure** $O(2, 2d-2)$ - H_{ab}.
ω, η, H **define Born geometry.** (FLM, '13, '14, '15, '16)
Their triple intersection gives the Lorentz group.
Thus **QM follows from non-locality (fundamental length/time) consistent with causality. (In general: dynamical causality - gravitized quantum theory.)**
Note: can be localized (local QFT possible!) in a particular phase space cell, but can't tell in which one (uncertainty principle)!

QG=Gravitization of Quantum Theory

How can fundamental length/time be consistent with Lorentz?
This is possible because of **relative (observer dependent) locality**. (Amelino-Camelia, Freidel, Kowalski-Glikman, Smolin) Different observers see different spacetimes (slices of modular spacetime). **Different spacetimes are in linear superposition, and so fundamental length/time is consistent with Lorentz.** (Similar to spin: the superposition of up and down spin gives the Bloch sphere which is consistent with rotation symmetry, even though spin is discrete.) (FLM)

QG=Gravitization of Quantum Theory

Generic quantum polarization (FLM, '16) - **modular polarization** (defined via the Zak transform). Given Schrödinger's $\psi_n(x)$

$$\psi_\lambda(x, \tilde{x}) \equiv \sqrt{\lambda} \sum_n e^{-2\pi i n \tilde{x}} \psi_n(\lambda(n + x)) \tag{43}$$

($x \equiv q/\lambda$, $\tilde{x} \equiv p/\epsilon$, so $[x, \tilde{x}] = i$, $\lambda\epsilon = \hbar$). Note, from the point of view of modular polarization, Schrödinger's polarization is very singular. Introduce $\mathbb{X}^A \equiv (x^a, \tilde{x}_a)^T$, so that $[\hat{\mathbb{X}}^a, \hat{\mathbb{X}}^b] = i\omega^{AB}$. We can write the translations operators in phase space covariantly $W_{\mathbb{K}} \equiv e^{2\pi i \omega(\mathbb{K}, \mathbb{X})}$, where $\mathbb{K}$ stands for the pair $(\tilde{k}, k)$ and $\omega(\mathbb{K}, \mathbb{K}') = k \cdot \tilde{k}' - \tilde{k} \cdot k'$. (Note W - **Aharonov-Bohm phases - prototypical example of modular variables**.)

QG=Gravitization of Quantum Theory

So far we have discussed covariant quantum phase space as an example of modular space, and so we are ready to discuss modular (quantum) spacetime. Consider (FKLM, '18) a **metaparticle** (mp) propagating in a modular space defined by Born geometry - ω, η, H. The metaparticle world-line action $S_{mp} = \int d\tau L_{mp}$ (canonical particle - $\mu \to 0$ and $\tilde{p} \to 0$)

$$L_{mp} = p_\mu \, \dot{x}^\mu + \tilde{p}^\mu \, \dot{\tilde{x}}_\mu + \lambda^2 p_\mu \, \dot{\tilde{p}}^\mu - \frac{N}{2} \left(p_\mu p^\mu + \tilde{p}_\mu \tilde{p}^\mu - m^2 \right) + \tilde{N} \left(p_\mu \tilde{p}^\mu - \mu \right), \tag{44}$$

where ω is in ("Berry-phase") $p_\mu \, \dot{\tilde{p}}^\mu$, and η in the diffeo constraint $p_\mu \tilde{p}^\mu = \mu$ and H in the Hamiltonian constraint $p_\mu p^\mu + \tilde{p}_\mu \tilde{p}^\mu = m^2$. **Dual spacetime** $\tilde{x}$, $[x, \tilde{x}] = i\lambda^2$, and **dual momentum space** $\tilde{p}$, $[p, \tilde{p}] = 0$. (Also, $[x, p] = i\hbar = [\tilde{x}, \tilde{p}]$.) **Spacetime** x **is quantum!**

QG=Gravitization of Quantum Theory

The metaparticle can be understood also as follows: If one second quantizes Schrödinger's $\psi(x)$ one naturally ends up with a quantum field operator $\hat{\phi}(x)$. Similarly, the second quantization of the modular $\psi_\lambda(x,\tilde{x})$ would lead to a modular quantum field operator $\hat{\phi}_\lambda(x,\tilde{x})$ (**modular fields - metafields**)

$$\hat{\phi}(x) \to \hat{\phi}_\lambda(x,\tilde{x}). \tag{45}$$

with $[x,\tilde{x}] = i\lambda^2$ - covariant non-commutative field theory. (FLM, '17) **Mixing of UV and IR. Not EFT. Contextual.** Classical spacetime label x of canonical QFT - choice of (classical spacetime) polarization in modular (quantum) spacetime with a contextuality parameter λ. **Average over dual spacetime $\tilde{x}$ to obtain classical evolution from the unitary evolution in the spacetime basis!**

QG=Gravitization of Quantum Theory

Quanta of canonical quantum fields $\phi(x)$ - particles (and their antiparticles).
Quanta of modular quantum fields $\phi_\lambda(x, \tilde{x})$ - metaparticles.
First prediction of modular spacetime approach to quantum theory - metaparticles! (FKLM)
(We will argue that dual particles, correlated to visible particles, represent dark matter.)
Note that if we turn on backgrounds $p \to p + \phi$ and $\tilde{p} \to \tilde{p} + \tilde{\phi}$. Thus we have **"dark matter" fields** $\tilde{\phi}(x)$ in the effective classical spacetime x description (after integrating over the dual spacetime $\tilde{x}$ - quantum measurement). **Visible ϕ and Invisible (dark matter) $\tilde{\phi}$ do not commute - $\tilde{\phi}$ describes fuzzy dark matter.**

QG=Gravitization of Quantum Theory

Dual "particles" (dual fields) - dark matter (to leading order in λ)

$$S_{eff} = -\int \sqrt{g(x)\tilde{g}(\tilde{x})}[R(x)+\tilde{R}(\tilde{x})+L_m(A(x,\tilde{x}))+\tilde{L}_{dm}(\tilde{A}(x,\tilde{x}))], \tag{46}$$

Here the A fields denote the usual Standard Model fields, and the $\tilde{A}$ are their duals, as predicted by the general (modular) formulation of quantum theory that is sensitive to the minimal length.
Note that we need to integrate over the dual space coordinates $\tilde{x}$ to get an effective description of **visible matter,** $A(x)$**, and dark matter,** $\tilde{A}(x)$, in classical x spacetime. (BHM, '21, '22)

QG=Gravitization of Quantum Theory

Dynamical geometry of dual spacetime - dark energy (to leading order in λ) [model for the equation of state of dark energy]

$$S_{eff} = -\int \sqrt{-g(x)}\sqrt{-\tilde{g}(\tilde{x})}[R(x) + \tilde{R}(\tilde{x}) + \ldots], \qquad (47)$$

In this leading limit, the $\tilde{x}$-integration in the first term defines the gravitational constant G_N, and in the second term produces a **positive cosmological constant constant**! (BHM, '21, '22) **In general: time-varying dark energy (HJKMT, '25)** - DESI! *Also, visible and dark matter degrees of freedom are correlated (via the minimal length λ) - origin (from fuzzy dark matter) of Milgrom's scaling (galaxies, clusters, superclusters) and Milgrom's acceleration* $a_0 \sim cH/(2\pi)$. ($\Lambda \sim H^2$.) (Edmonds, Erlich, Minic, Takeuchi).

QG=GQ - Explicit Realization

Explicit realization in terms of a chiral phase-space reformulation of the bosonic string, the "**metastring**," (FLM, '13, '14, '15) - also a non-perturbative proposal (BHM '21, '22) of QG (matrix model-like, time-asymmetric (?), $\partial_\sigma \cdot \equiv [\hat{\mathbb{X}}, \cdot]$, where $\hat{\mathbb{X}}$ matrix comes from modular world-sheet) - spacetime/matter quanta

$$S_{\rm str}^{\rm ch} = \int \mathrm{d}\tau \mathrm{d}\sigma \left[\partial_\tau \mathbb{X}^a \left(\eta_{ab}(\mathbb{X}) + \omega_{ab}(\mathbb{X})\right) - \partial_\sigma \mathbb{X}^a H_{ab}(\mathbb{X})\right] \partial_\sigma \mathbb{X}^b, \tag{48}$$

where $\mathbb{X}^a \equiv (X^a/\lambda, \tilde{X}_a/\lambda)^T$ are coordinates on phase-space like (doubled) target spacetime and η, H, ω are all dynamical. $x^a, \tilde{x}_a$ come from the left and right moving modes of the bosonic string,

$$x^a \equiv x_L^a + x_R^a, \quad \tilde{x}^a \equiv \tilde{x}_L^a - \tilde{x}_R^a \tag{49}$$

In the context of a flat metastring we have constant η_{ab}, H_{ab} and ω_{ab} (zero ω_{ab} - connection to double field theory)

$$\eta_{ab} = \begin{pmatrix} 0 & \delta \\ \delta^T & 0 \end{pmatrix}, \quad H_{ab} = \begin{pmatrix} h & 0 \\ 0 & h^{-1} \end{pmatrix}, \quad \omega_{ab} = \begin{pmatrix} 0 & \delta \\ -\delta^T & 0 \end{pmatrix}, \tag{50}$$

The standard Polyakov action is obtained when setting $\omega_{ab} = 0$ and integrating out the $\tilde{x}_a$ (h_{ab}(X) - gravity; zero beta function for h_{ab} - Einstein's equations)

$$S_P = \int \mathrm{d}\tau \mathrm{d}\sigma \gamma^{\alpha\beta} \partial_\alpha X^a \partial_\beta X^b h_{ab}(X) + \dots \tag{51}$$

The triplet (ω, η, H) define the Born geometry (FLM, '13, '14) and the metastring propagates in modular, not classical, spacetime. Recall: the *space* of commuting subalgebras of the Heisenberg algebra, $[\hat{x}, \hat{\tilde{x}}] = i\lambda^2$, defines modular spacetime (FLM, '15, '16)

The new feature in the metastring formulation of the bosonic string is intrinsic non-commutativity and so there is a new Heisenberg algebra (vertex operators reps of Weyl-Heisenberg - no cocycles)

$$[\mathbb{X}^a, \mathbb{X}^b] = i l_s^2 \omega^{ab} \implies [X^a, \tilde{X}^b] = i\delta^{ab} l_s^2 \tag{52}$$

$[\delta q \sim G_N \delta p \to \delta q \sim G_N \delta \tilde{p} \sim G_N \frac{1}{\delta \tilde{q}} \to \delta q \delta \tilde{q} \sim G_N \to [q, \tilde{q}] \sim i l_P^2]$ as well as the standard commutators (with $[\Pi, \tilde{\Pi}] = 0$)

$$[\mathbb{X}^a, \mathbb{P}_b] = i\hbar\delta^a_b \implies [X^a, \Pi_b] = i\delta^a_b\hbar, \quad [\tilde{X}^a, \tilde{\Pi}_b] = i\delta^a_b\hbar \tag{53}$$

Note, if Kalb-Ramond B_{ab} (axion) constant but non-zero, dual coordinates do not commute! In general - non-associativity [SM] (FLM,'17.) **Zero modes of the metastring - metaparticles** (FKLM, '18, '21) - little rigid strings. (Each Standard Model (SM) particle has a correlated "dual" - dark matter pheno (BHM, '20, '21).)

How could we "see" modular spacetime?

Instead of scattering particles, entwine them!
Vertex operators V_K (plane waves - asymptotic particle states) have co-cycles in the Polyakov string if we assume that $[x, \tilde{x}] = 0$

$$V_{\mathbb{P}} V_{\mathbb{P}'} = e^{i(p\tilde{p}' - \tilde{p}p')} V_{\mathbb{P}'} V_{\mathbb{P}} \tag{54}$$

The cocycle factor $e^{i(p\tilde{p}' - \tilde{p}p')}$ indicates the fundamental non-commutativity of x and $\tilde{x}$.
Can this entwining of particles be measured?
(Here we are really talking about the "R-matrix", in the sense of "swapping of particles", instead of the S-matrix - "scattering of particles".)
[2d reduction (high temperature behavior of the metastring), "asymptotic silence of QG" - planar 4-jets at a couple of TeV at LHC (Nina Ilic, Dejan Stojkovic, DM, et al)]

QG=GQ - Explicit Realization

The metastring has **dynamical Born geometry**, (FLM, '14, '15) $\omega_{ab}(\mathbb{X}), \eta_{ab}(\mathbb{X}), H_{ab}(\mathbb{X})$, but Born geometry is the geometry of the modular spacetime formulation of quantum theory.
Thus **by making Born geometry dynamical we can "gravitize quantum theory"** (that is, **make the geometry of quantum theory dynamical**)! (FLM)
Also, metastring is a theory of quantum gravity, and so we arrive at **"QG = gravitized quantum theory". - Triple and higher order interference**
This reasoning is "top-down".
(Note, classical GR gravitizes all of classical physics!)

Consider particle interactions as $0+1$ quantum gravity.
Quantum field theory $= 0+1$ quantum gravity/cosmology!
Example: $g\phi^3$ theory. Classical equations:

$$(\partial^2 + m^2)\phi + g\phi^2 = 0 \tag{55}$$

Note ϕ - wave-function of $0+1$ universes.
Thus the above equation: non-linear Wheeler-DeWitt equation.
Interaction vertex: **topology change**.
Classical spacetime viewpoint: decay of a particle (Born rule or S-matrix)
QG=GQ viewpoint from $0+1$ universes (particles) - **triple correlation!** [Gravitons: **n-correlations; n = 3,4,5...** - classical spacetime hides higher order correlations.] **Same for the pants diagram for strings! It represents intrinsic triple interference.**

QG=GQ - Explicit Realization

Finally: *Matter is granular and cuttable*: it consists of fermions that are held together through interactions that are mediated by bosons (the spin-statistics theorem of local QFT).
In order to extend this atomic picture to spacetime we note the fundamental difference between spacetime and matter: *spacetime is extended and non-cuttable*.
The claim here is that **spacetime quanta obey infinite statistics** (*quantum distinguishable, or quantum Boltzmann statistics,* $a_i a_j^\dagger = \delta_{ij}$ - Cuntz free algebra and non-commutative probability theory applicable to matrix models) and are *held together by higher order quantum correlations responsible for higher order interference effects.* (Spacetime Avogadro number $N \sim 10^{31}$ and the vacuum energy formula $\delta \sim \sqrt{l l_P}$.) **Gravitational Brownian motion via gravitational interferometry.** (Zurek et al).

Table of Contents

Summary

Quantum gravity (QG) = Gravitization of quantum theory (GQ)
Gravitization of quantum theory = dynamical geometry of quantum theory (dynamical Born geometry)
In principle, not only double, but also triple and higher order interference allowed! "Smoking gun experiment".
Quantum spacetime = modular spacetime (geometry of quantum theory). Spacetime and its dual do not commute! (Measurement!)
Consequences: Particles (visible matter) are singular limits of metaparticles - correlated particles and dual (dark) particles
Quantum fields and dual (dark) quantum fields do not commute (fuzzy dark matter). Dark energy - curvature of dual spacetime
Metaparticles - zero modes of the metastring (non-commutative string in a dynamical geometry of modular spacetime) (QG=GQ)

Outlook

Phenomenological Implications of QG=GQ and future work:

- Cosmological constant (CC) as a guiding empirical quantum gravity phenomenon - (also, gravitational interferometry and QG Brownian motion; LHC signals - 2d reduction)
- Metaparticles (zero modes of the metastring) and dark matter (entangled/correlated SM/dual (DM) particles) - fuzzy DM
- Dark energy (CC plus time variation) as the curvature of the dual spacetime (CC naturally small) - DESI, etc.
- Gravitational wave "echoes" - $N \sim l^2/l_P^2$ for black holes (quantum chaos - quantum scars?).
- Dynamical Born geometry, "gravitizing the quantum," look at triple (and higher order) quantum interference in QG!
- The CC, the Higgs mass and SM fermion masses and mixing. (**Bounds! - Cosmological Attractor?**)