Singularity resolution in infinite derivative gravity theories

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Happy Birthday, Branko!

Mainly based on arXiv:2404.07925 and arXiv:2412.02678 in collaboration with Chenxuan Li and Anna Tokareva,

past works with Alexei Starobinsky and works in progress

with O.Melichev, A.Naskar, L.Rachwal, A.Tokareva and my students

Breakdown of the problem

UV complete gravity – already a challenge for more than a century

- **Many attempts, no complete satisfaction yet**

Infinite derivatives

- **General considerations and, for example, Asymptotic Safety suggest infinite derivative Lagrangians**

Strings

- **Strings and especially string field theory strongly suggest non-local interactions in the form of infinite-derivative form factors**

Aref'eva, Barvinsky, Biswas, Dragovich, Koivisto, Krasnikov, Kuz'min, Mazumdar, Modesto, Percacci, Platania, Saueressig, Sen, Siegel, Shapiro, Tomboulis, Weinberg, Witten, Zwiebach, . . .

Some old references

- **Classic one:**

  M. Ostrogradski, Mem. Ac. St. Petersburg, VI 4, 385–517 (1850)

- **Mathematical:**

- H.T. Davis, Ann. of Math. 2, no. 4, 686–714 (1931)
- H.T. Davis, The Theory of Linear Operators from the Standpoint of Differential Equations of Infinite Order (Indiana, the Principia Press, 1936)
- R.D. Carmichael, Bull. Amer. Math. Soc. 42, 193–218 (1936)
- L. Carleson, Math. Scand. 1, 31–38 (1953)

- **Physical:**

- A. Pais and G.E. Uhlenbeck, Phys. Rev. 79, 145–165 (1950)

"Convergence" (renormalizability), "definite norm" (unitarity) and causality – cannot be achieved simultaneously. Fine, but what if violation of microcausality is hidden under the uncertainty scale? de Rham, Tokareva, Tolley, ...

Action to study [1602.08475, 1606.01250, 1711.08864]

$$S = \int d^4x\sqrt{-g}\Bigg(\frac{M_P^2 R}{2} - \Lambda$$
$$+\frac{\lambda}{2}\left(R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2 R^{\mu\nu} + R_{\mu\nu\lambda\sigma}\mathcal{F}_4(\Box)R^{\mu\nu\lambda\sigma}\right)\Bigg)$$

Here $\mathcal{F}_{1,4}(\Box) = \sum_{n\geq 0} f_{1,4_n}\Box^n$ with all $f_{1,4_n}$ constants

$\Box$ enters in a combination $\Box/\mathcal{M}_s^2$ where the mass parameter is the non-locality scale. We put $\mathcal{M}_s = 1$ for a while.

This is the most general action (still redundant, $\mathcal{F}_2$ can be zero in $D=4$ or a constant in $D>4$) to study linear perturbations around MSS.

Consistency requires $\mathcal{F}_1(\Box) + \frac{1}{3}\mathcal{F}_4(\Box) = 0$ or an exponent of an entire function (around $D=4$ Minkowski with $\mathcal{F}_2 = 0$).

We name it Analytic Infinite Derivative (AID) gravity.

Covariant spin-2 propagator on MSS:

$$S_2 = \frac{1}{2}\int d^4x\sqrt{-\bar{g}}\; h^{\perp}_{\nu\mu}\left(\bar{\Box} - \frac{\bar{R}}{6}\right)\left[\mathcal{P}(\bar{\Box})\right] h^{\perp\,\mu\nu}$$

$$\mathcal{P}(\bar{\Box}) = 1 + \frac{2}{M_P^2}\lambda f_{1_0}\bar{R} + \frac{2}{M_P^2}\lambda\mathcal{F}_4\left(\bar{\Box} + \frac{\bar{R}}{3}\right)\left(\bar{\Box} - \frac{\bar{R}}{3}\right)$$

$$\rightarrow e^{2\omega(\bar{\Box})}$$

We require $\mathcal{P}(\bar{\Box}) = e^{2\omega(\bar{\Box})}$ and $\omega(\bar{\Box})$ must be an entire function to avoid new poles.

The Stelle's case (and any finite degree polynomial $\mathcal{F}_4(\bar{\Box})$) results in ghost poles.

Infinite derivative gravity theories in short

- Graviton propagator in general is modified to

$$\Pi = e^{2\omega(k^2)}\Pi_{GR} \sim \frac{e^{2\omega(k^2)}}{k^2}$$

  $\omega(k^2)$ must be an entire function. It must grow logarithmically along the real axis, hence its order must be infinite.
- We thus must have an infinite number of derivatives
- Wick rotation is a problem but it got a resolution thanks to Pius, Sen, and also [arxiv:2103.01945]
- Theory is renormalizable and unitary.
- Full propagator yet to be computed.
- Many interesting solutions can be accommodated.
- In particular, Starobinsky inflation can be explicitly embedded.

Action again

$$S = \int d^4x \sqrt{-g} \left(\frac{M_P^2 R}{2} + \frac{\lambda}{2} \left(R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\lambda\sigma}\mathcal{F}_4(\Box)R^{\mu\nu\lambda\sigma} \right) \right)$$

If $\mathcal{F}_4 \neq 0$ than a Schwarzschild BH is not a solution.

Even if $\mathcal{F}_4 = 0$ we claim it is not!

WHY?

Schwarzschild BH: to be or not to be?

We cannot substitute the Schwarzschild metric like in GR as we need to give a meaning for instance for $R(r=0)$

So, we take

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2 d\Omega^2$$

Regularization in the Schwarzschild case:

$$A(r) = 1 - \frac{2GM}{r} \to A(r) = 1 - \frac{2GM}{r}\tilde{A}(r,\alpha), \;\; \tilde{A} = e^{-\alpha/r^p}$$

We plug a regularized function in EOM-s and compute

$$\int d^3x \sqrt{-g} T^{\mu}_{\mu} = E$$

which is related to the energy of the object. In static case it is related to its mass.

What is a BH mass?

What we compute is $E = \int d^3x\sqrt{-g}(T^i_i + T^0_0)$.

Tolman mass is defined as $M_T = \int d^3x\sqrt{-g}(T^i_i - T^0_0)$.

ADM mass is a coefficient of $1/r$ term in a series expansion of g_{rr} metric component at infinity divided by $2G$, or equivalently $M_{ADM} = -\int d^3x\sqrt{-g}T^0_0$.

Thus E is nothing but $M_T - 2M_{ADM}$ and should correspond to $-M_{ADM}$.

It is naturally expected to be a finite quantity.

To simplify computations we actually compute

$$\lim_{\Delta t\to\infty} \frac{1}{2\Delta t}\int_{-\Delta t}^{\Delta t} dt d^3x\sqrt{-g}T$$

Schwarzschild BH in higher-derivative theories

Computing E we schematically yield

$$-E = M - 4\pi\lambda(E_0 + E_1 + E_2 + \dots)$$

Here E_n corresponds to $\Box^n$ and for $p = 1$

$$E_0 \sim 1/\alpha^3, \quad E_1 \sim 1/\alpha^6 + 1/\alpha^5, \quad \dots$$

E_0 comes from $\mathcal{F}(\Box) \sim \log(\Box)$

The above series *can* converge if it is alternating with rapidly falling coefficients. Example

$$\sum_{k\geq 0} \frac{(-1)^k}{k!\alpha^k} = e^{-1/\alpha} \xrightarrow{\alpha\to 0} 0$$

BH results briefly and what about micro-BH?

- Regularization approach is motivated by a collapse consideration. You must be able to form a BH starting with a regular matter distribution.
- Regularization of a Schwarzschild BH can be removed only in 2 and 4 derivative gravity. Any higher (finite) derivative gravity cannot have this solution.
- Infinite derivative case results in infinitely many terms like $1/\alpha^n$ and in principle this sum may have a good $\alpha \to 0$ limit. The order of $\mathcal{F}(\Box)$ must be less than 3/2
  BUT for a viable propagator of a UV complete unitary gravity the order of $\mathcal{F}(\Box)$ must be ∞
- We thus must accept that a UV complete gravity not only resolves the BH singularity but also limits the micro-BH mass from below to $\mathcal{M}_s$ which obeys $M_{inf} \ll \mathcal{M}_s < M_P$

What about other types of singularities?

- Regularization approach can be applied to any singular behavior.
- We can attack a cosmological singularity, for example.
- For a FLRW metric $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$ we can regularize the scale factor $a(t)$, compute the action and study a limit without regularization
- Infinite action will indicate an improbable configuration since it will correspond to a heavily oscillating expression in the path integral $Z = \int d\varphi e^{iS(\varphi)}$
- An observation is that known physically relevant singular space-times result in an infinite action in AID gravity.

Conclusions

- **A class of analytic infinite derivative (AID) theories has been considered targeting the goal of constructing a UV complete and unitary gravity. These models have clear connection with SFT.**
- **This gravity model features many nice properties, like native embedding of the Starobinsky inflation, finite Newtonian potential at the origin, presence of a non-singular bounce, etc.**
- **We argue that these theories disregard singular BH solutions on the example of Schwarzschild BH.**
- **We extend this statement to other types of singularities.**

Future directions

- **Other BH solutions (charged, extremal, rotating) should be analyzed.**
- **BH regularity as a given feature implies that QNM may be modified.**
- **QNM will not test the interior of a BH as such, but higher derivatives in the action will result in new QNM shapes which is a very interesting way to support the idea that a UV complete gravity resolves BH singularities.** [arxiv:2412.02678]
- **Cosmological and other singularities should be studied systematically using the finite action argument.** **Work in progress**

Thank you for listening!

Let it be a Non-local scalar field [arxiv:2103.01945]

Consider Analytic Infinite Derivative (AID) scalar field action:

$$L = \frac{1}{2}\phi(\Box - m^2)f^{-1}(\Box)\phi - V(\phi)$$

We demand the form-factor to be an exponent of an entire function $\sigma(z)$

$$f(z) = \exp(2\sigma(z))$$

This is required to have no extra poles in the perturbative vacuum.

We also normalize it as $f(0) = f(m^2) = 1$ to preserve the local answers in the IR limit.

Non-local scalar field, continued

Several arguments to consider the above action:

- **It naturally appears in SFT and in p-adic strings**
- **It was proven to be unavoidable in order to build unitary and renormalizable diffeomorphism invariant gravity**
- **This construction can make any arbitrary potential renormalizable**
- **Surely, some other benefits**

Namely, we can adjust the fall rate of the propagator for large momenta by choosing the form-factor. Power-counting convergence requires the fall faster than $\sim 1/p^2$.

New excitations – Half of them are ghosts!

Linearization around a background solution ϕ_0:

$$L = \frac{1}{2}\psi\left[(\Box - m^2)f^{-1}(\Box) - V''(\phi_0)\right]\psi$$

Let's assume $V''(\phi_0) = v \approx \text{const} \neq 0$.

- **In general there is an infinite number of new excitations with perhaps complex conjugate masses squared**
- **The kinetic operator is again an entire function and obeys the Weierstrass decomposition**

$$(\Box - m^2)f^{-1}(\Box) - v^2 \sim \prod_i(\Box - \mu_i^2)e^{\sigma_v(\Box)}$$

- **Each μ_i corresponds to a mass of a distinct excitation.**