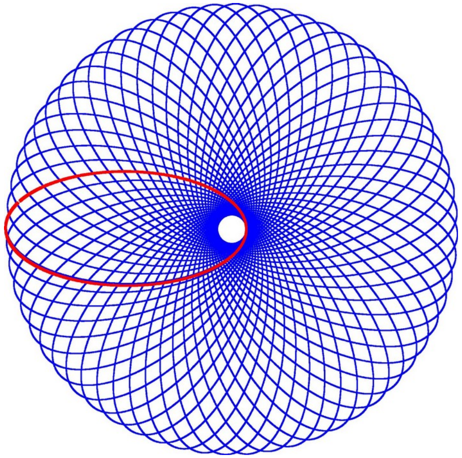

Possible influence of a fifth force on stellar orbits around the Galactic Center

Predrag Jovanović[1], Duško Borka[2], Vesna Borka Jovanović[2]

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[2]Department of Theoretical Physics and Condensed Matter Physics (020), Vinča Institute of Nuclear Sciences, P.O. Box 522, 11001 Belgrade, Serbia

Outline

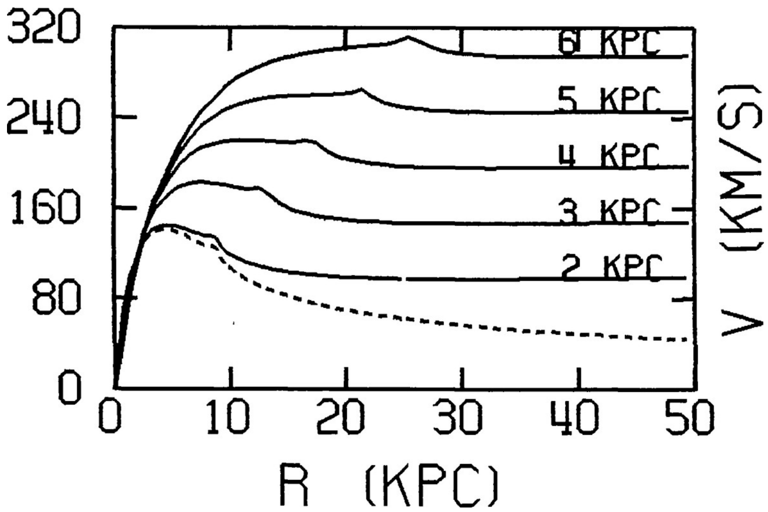
1. Yukawa gravity with fifth force
2. Graviton mass and speed of gravity
3. $f(R)$ gravity and Yukawa-like correction to the gravitational potential
4. Brief overview of the existing observational constraints on: Yukawa gravity with fifth force, mass of graviton and speed of gravity
5. Observed stellar orbits around Sgr A* at Galactic Center
6. Detection of Schwarzschild precession (and possible small deviations from GR prediction) in the orbit of S2 star around Sgr A* by GRAVITY Collaboration in 2020
7. Our previous and recent results, obtained in collaboration with Prof. Alexander F. Zakharov (Russia) and Prof. Salvatore Capozziello (Italy):
   - Constraining the fifth force and graviton mass by analysis of the observed stellar orbits in Yukawa gravity
   - Improved constraints from the observed Schwarzschild precession in the S2 star orbit
8. Conclusions

Modified gravity with fifth force

- Concept of fifth force emerges in the attempts to unify gravity with the other forces
- It is able to produce nearly flat rotation curves of spiral galaxies without need for dark matter hypothesis, and it can also mimic dark energy effects
- **Fifth force**: an additional effective repulsive (anti-gravity) force which arises in the weak field limit of massive gravity theories and some Extended Theories of Gravity (ETG) in the form of exponential **Yukawa-like correction** to the Newtonian gravitational potential:

$$\Phi\left(r\right)=-\frac{G_{\infty}M}{r}\left(1+\alpha\, e^{-r/\lambda}\right)$$

$$G_0=G_{\infty}\left(1+\alpha\right)\quad\wedge\quad\alpha\sim-1$$



- α - strength of the fifth force, λ - its range
- λ corresponds to the Compton wavelength of graviton with small mass m_g: $\lambda_g=\dfrac{h}{m_g\,c}$
- Phenomenological assumption for gravitational potential in the case of massive graviton (Will, 1998, PRD, 57, 2061):

$$\Phi\left(r\right)=-\frac{GM}{r}\,e^{-\frac{r}{\lambda}}$$

1984A&A...136L..21S

Astron. Astrophys. 136, L21–L23 (1984)

ASTRONOMY AND ASTROPHYSICS

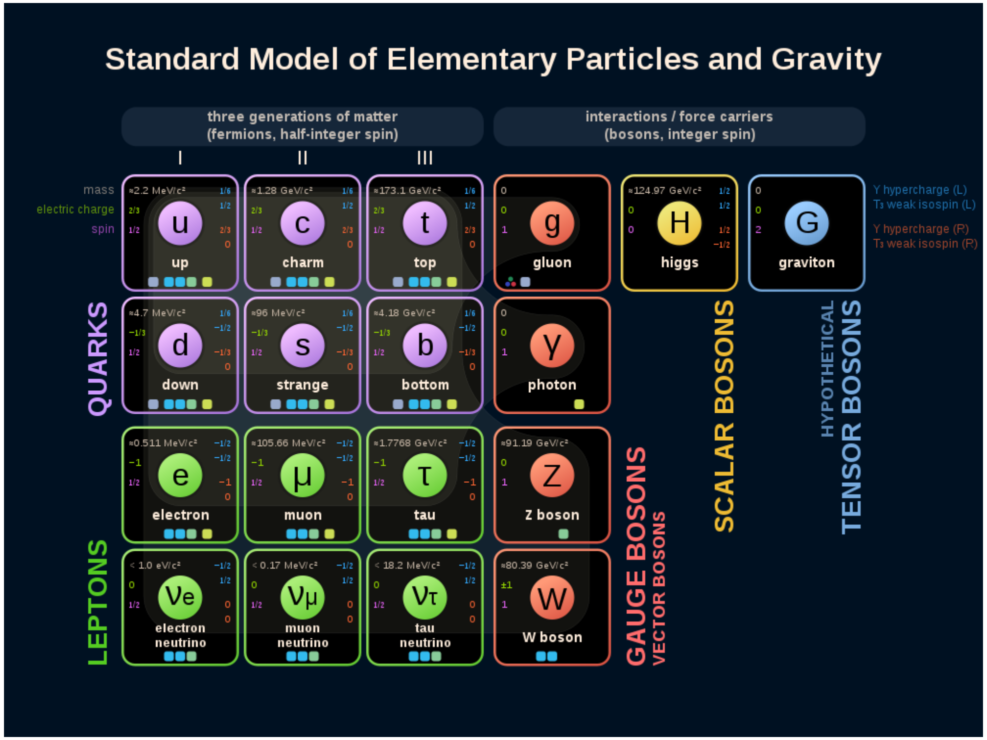
Letter to the Editor

Anti-gravity and galaxy rotation curves

R. H. Sanders

Graviton mass and speed of gravity

- Graviton:
  - Spin: 2 (tensor boson)
  - Electric charge: 0 (neutral)
- General Relativity (GR): graviton is massless and travels along null geodesics (like photon), i.e. at the speed of light c
- Theories of massive gravity (indroduced by Fierz & Pauli, 1939, RSPSA, 173, 211): gravitation is propagated by a massive field (i.e. by graviton with small, nonzero mass m_g)
- **Modified dispersion relation** (Will, 1998, PRD, 57, 2061): $E^2 = p^2c^2 + m_g^2c^4$



- Massive graviton propagates at an energy (or frequency) dependent speed v_g:

$$v_g^2/c^2 \equiv c^2p^2/E^2 \Rightarrow \boxed{v_g^2/c^2 = 1 - m_g^2c^4/E^2 = 1 - h^2c^2/(\lambda_g^2E^2) = 1 - c^2/(f\lambda_g)^2}$$

- v_g can be measured from the arrival and emission time differences Δt_a and Δt_e between a GW and an EW signal emitted from the same source at a distance D:

$$1 - \frac{v_\mathrm{g}}{c} = 5\times10^{-17}\left(\frac{200\ \mathrm{Mpc}}{D}\right)\left(\frac{\Delta t_\mathrm{a} - (1+z)\Delta t_\mathrm{e}}{1\ \mathrm{s}}\right),$$

where z is the redshift of the source

$f(R)$ gravity and Yukawa-like correction

- Gravitational potential with a Yukawa correction can be obtained in the Newtonian limit of any analytic f(R) gravity model (Capozziello et al. 2014, PRD, 90, 044052)
- Action for $f(R)$ gravity: $\mathcal{S} = \int d^4x\sqrt{-g}\left[f(R) + \mathcal{X}\mathcal{L}_m\right], \quad \mathcal{X} = \frac{16\pi G}{c^4}$
- 4th-order field eqs: $f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - f'(R)_{;\mu\nu} + g_{\mu\nu}\Box f'(R) = \frac{\mathcal{X}}{2}T_{\mu\nu}$
- Trace: $3\Box f'(R) + f'(R)R - 2f(R) = \frac{\mathcal{X}}{2}T$
- Analytic Taylor expandable function $f(R)$:

$$f(R) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} R^n = f_0 + f_1 R + \frac{f_2}{2}R^2 + \ldots \quad \Rightarrow$$

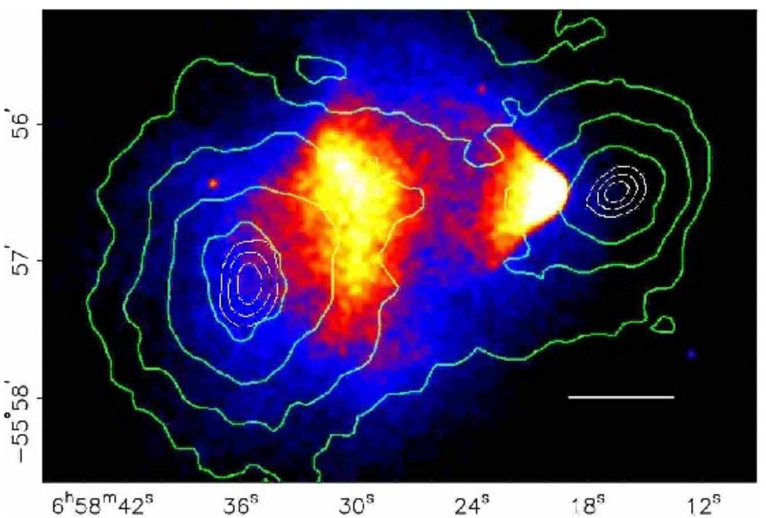
- Metric: $ds^2 = \left[1 + \frac{2\Phi(r)}{c^2}\right] c^2dt^2 - \left[1 - \frac{2\Psi(r)}{c^2}\right] dr^2 - r^2 d\Omega^2$
- **Yukawa-like correction** to the gravitational potential in the weak field limit:

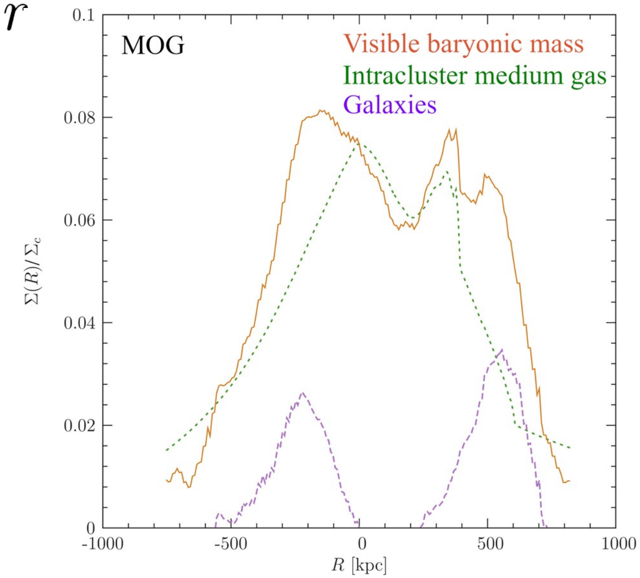
$$\boxed{\Phi(r) = -\frac{GM}{(1+\delta)r}\left(1 + \delta e^{-\frac{r}{\Lambda}}\right)} \qquad \Psi(r) = \frac{GM}{(1+\delta)r}\left[\left(1 + \frac{r}{\Lambda}\right)\delta e^{-\frac{r}{\Lambda}} - 1\right]$$

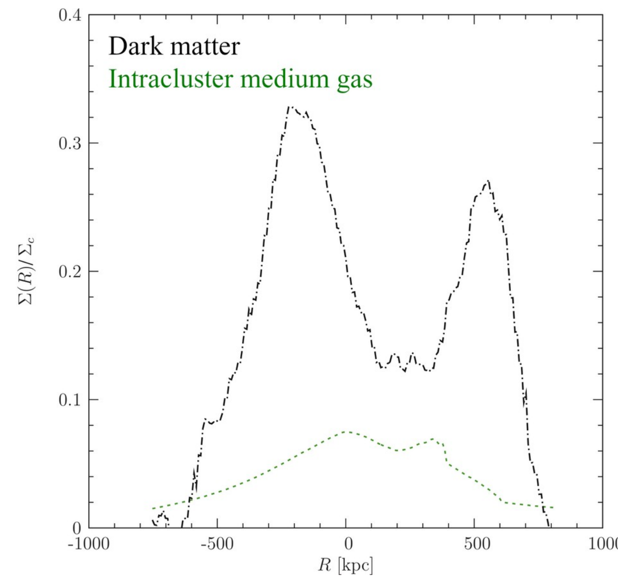
$$\Lambda^2 = -f_1/f_2 \quad \wedge \quad \delta = f_1 - 1$$

Yukawa gravity and the "Bullet cluster"

- Besides the flat rotation curves of spiral galaxies, Yukawa gravity can also explain the "Bullet cluster" without dark matter hypothesis
- The surface density reconstructed from weak lensing in Yukawa gravity in the absence of dark matter (DM) accounts for the 8σ spatial offset between the observed X-ray surface density and that reconstructed in GR in the presence of DM
- MOG: $\Phi\left(r\right) = -\frac{G_\infty M}{r} + \sigma \frac{e^{-\mu r}}{r}$







Mon. Not. R. Astron. Soc. **382,** 29–47 (2007)

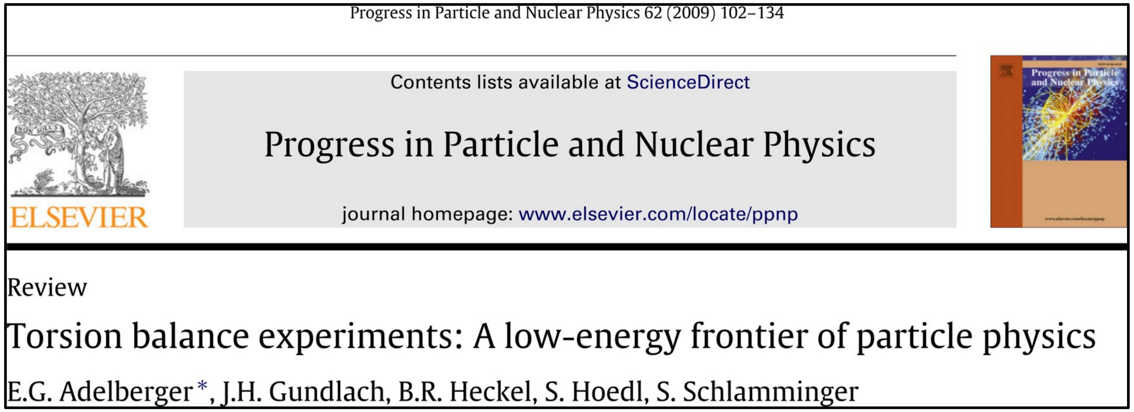
The Bullet Cluster 1E0657-558 evidence shows modified gravity in the absence of dark matter

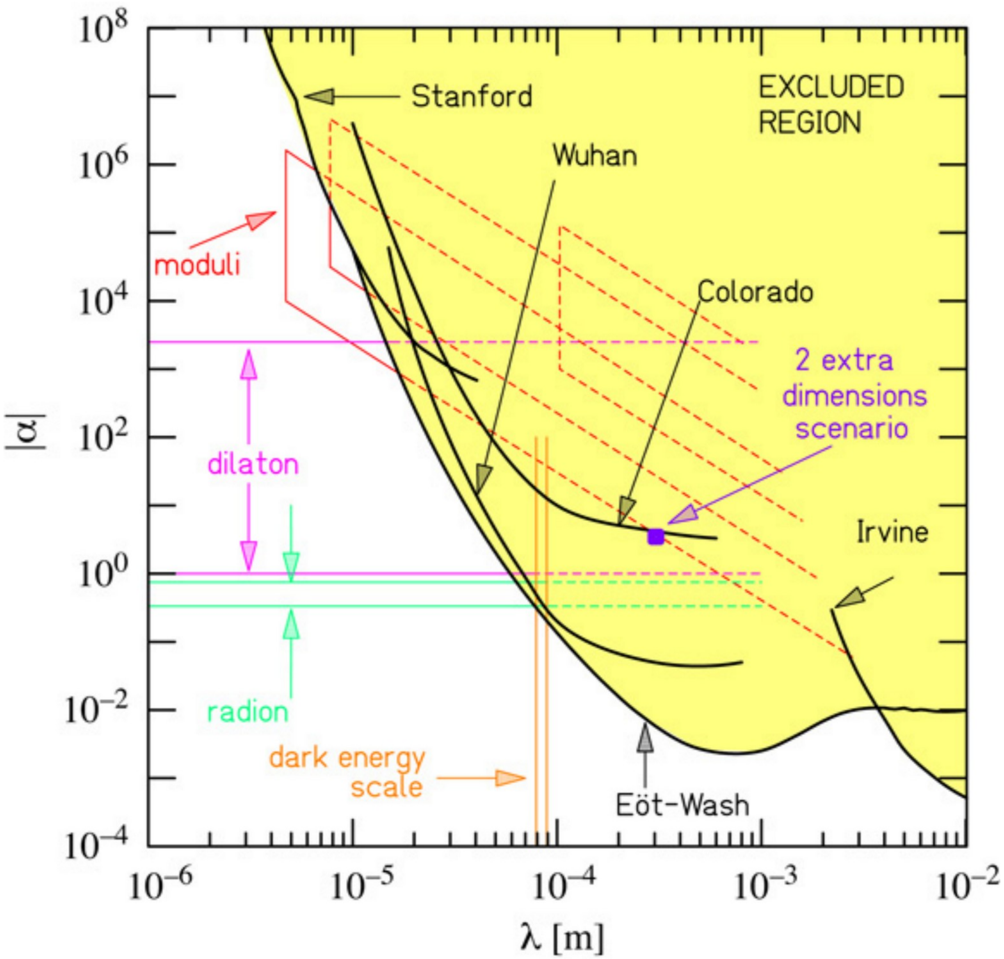
J. R. Brownstein[1★] and J. W. Moffat[2★]

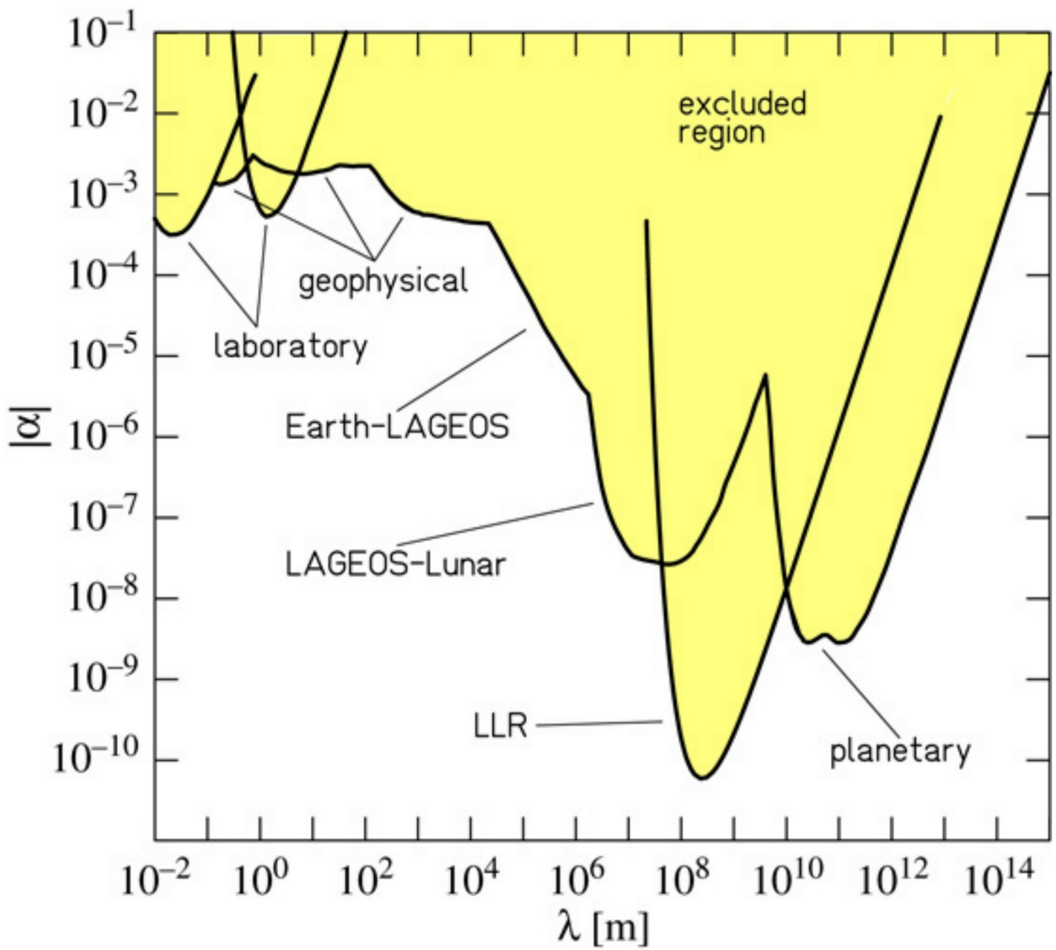
Experimental constraints on fifth force

- Experimental constraints to additional Yukawa gravitational interaction between masses m_1 and m_2:

$$V(r) = -G_N \frac{m_1 m_2}{r} \left(1 + \alpha\, e^{-r/\lambda}\right)$$


Progress in Particle and Nuclear Physics 62 (2009) 102–134
Contents lists available at ScienceDirect
Progress in Particle and Nuclear Physics
journal homepage: www.elsevier.com/locate/ppnp
ELSEVIER
Review
Torsion balance experiments: A low-energy frontier of particle physics
E.G. Adelberger *, J.H. Gundlach, B.R. Heckel, S. Hoedl, S. Schlamminger





LIGO constraints on fifth force and graviton mass

- Yukawa type correction with characteristic length scale λ_g: $\varphi(r) = \frac{GM}{r}\left(1 - e^{-r/\lambda_g}\right)$
- LIGO bound from GW150914:

$$\boxed{\lambda_g > 1.6 \times 10^{13}\ \mathrm{km}} \quad \Rightarrow$$

$$\boxed{m_g \leq 1.2 \times 10^{-22}\ \mathrm{eV}/c^2}$$

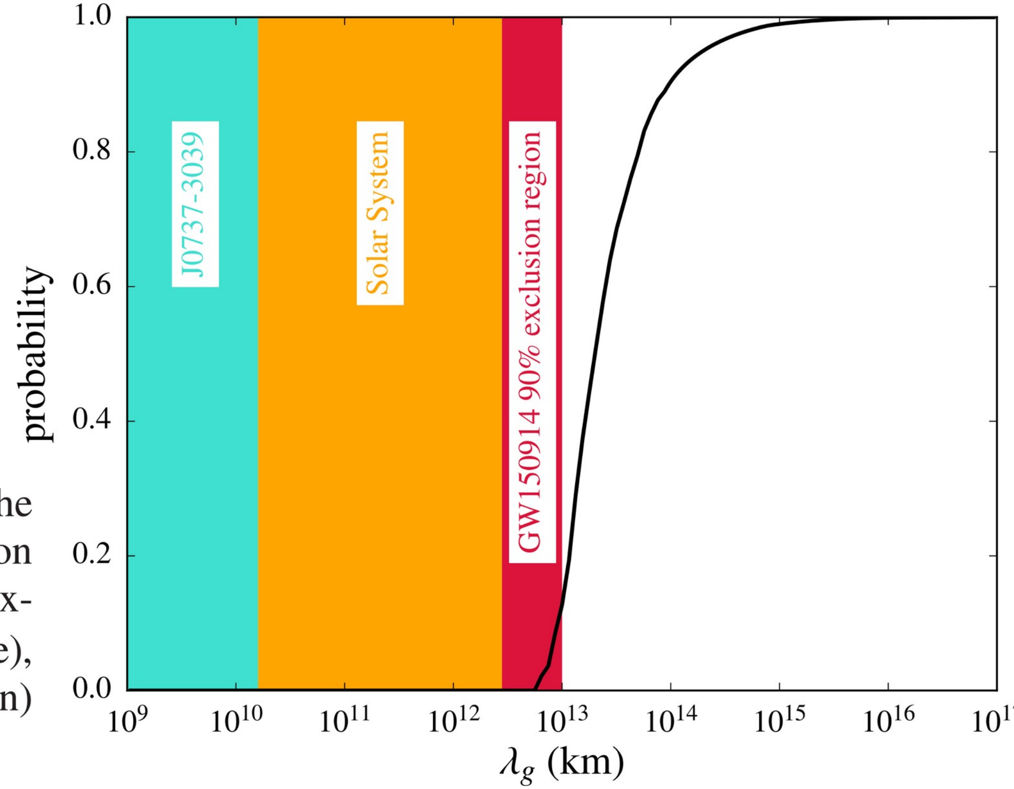


FIG. 8. Cumulative posterior probability distribution for λ_g (the black curve) and exclusion regions for the graviton Compton wavelength λ_g from GW150914. The shaded areas show exclusion regions from the double-pulsar observations (turquoise), the static Solar System bound (orange), and the 90% (crimson) region from GW150914.

Selected for a Viewpoint in *Physics*

PRL **116,** 221101 (2016) PHYSICAL REVIEW LETTERS week ending 3 JUNE 2016

Tests of General Relativity with GW150914

B. P. Abbott *et al.**

(LIGO Scientific and Virgo Collaborations)

Observational constraints on speed of gravity

THE ASTROPHYSICAL JOURNAL LETTERS, 848:L13 (27pp), 2017 October 20 https://doi.org/10.3847/2041-8213/aa920c

Gravitational Waves and Gamma-Rays from a Binary Neutron Star Merger: GW170817 and GRB 170817A

LIGO Scientific Collaboration and Virgo Collaboration, *Fermi* Gamma-ray Burst Monitor, and INTEGRAL

- GW and γ-rays from a binary neutron star merger in the galaxy NGC 4993 at $z \approx 0.01$ and $D = 26$ Mpc:

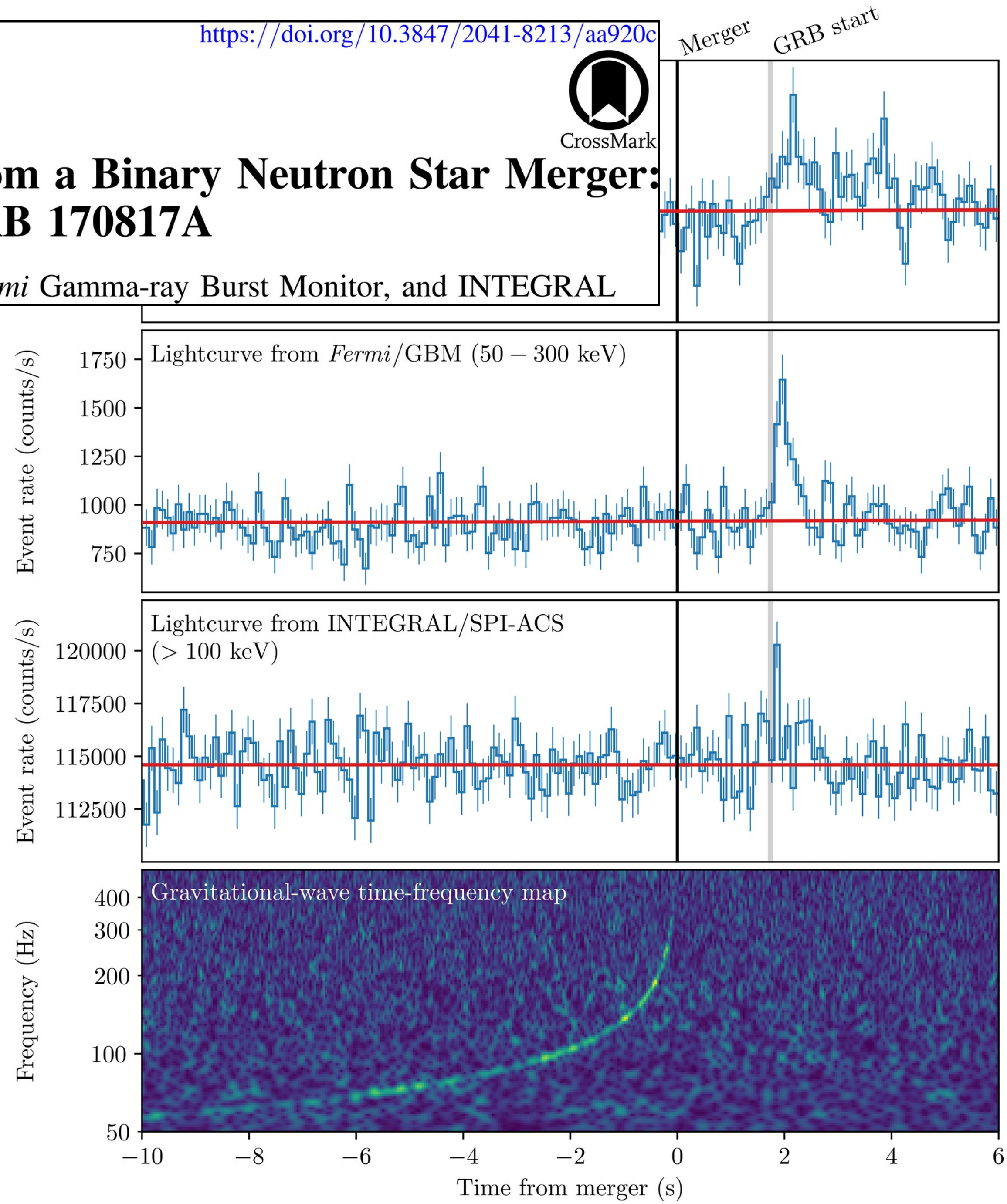
$$\Delta t \equiv \Delta t_{\mathrm{a}} - (1+z)\Delta t_{\mathrm{e}} \;\Rightarrow$$

$$\text{Case 1}: \Delta t = \Delta t_{\mathrm{a}} = 1.74\,\mathrm{s}$$

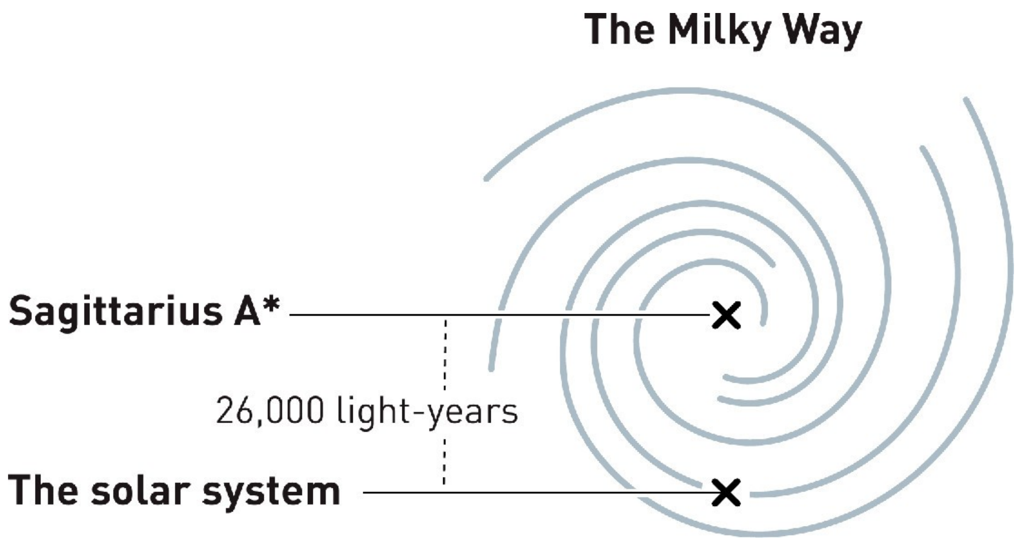
$$\boxed{\frac{v_{\mathrm{g}}}{c} - 1 \leq +7 \times 10^{-16}}$$

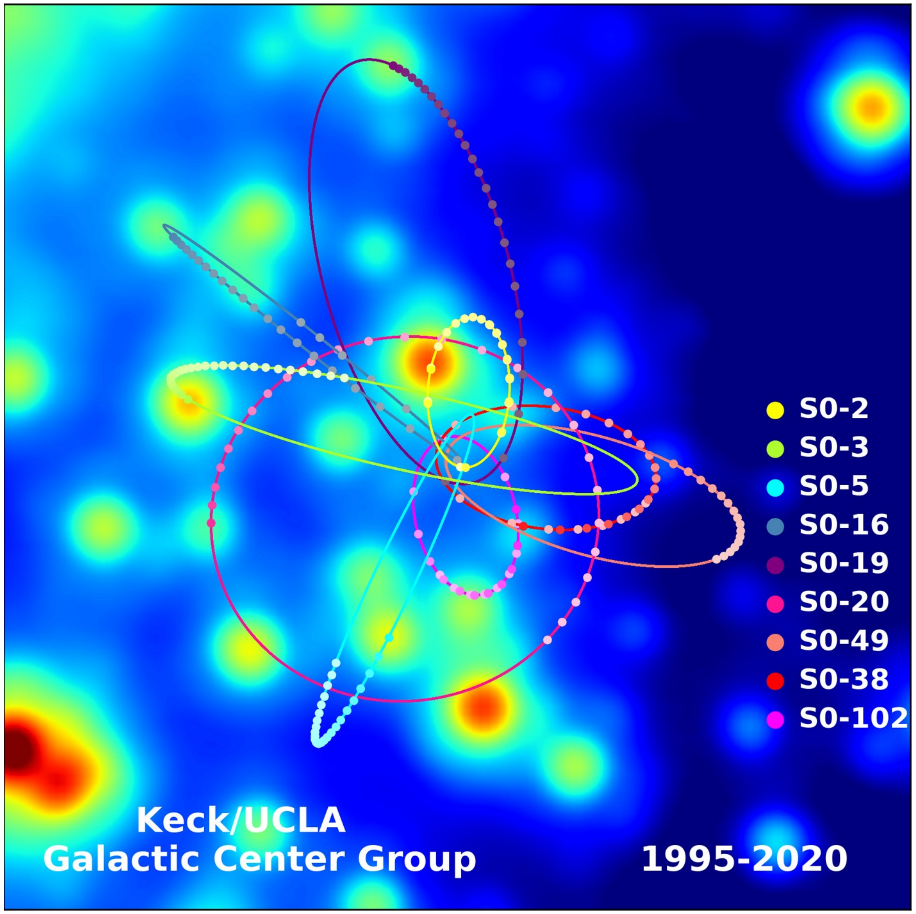
$$\text{Case 2}: \Delta t = 10\,\mathrm{s}$$

$$\boxed{\frac{v_{\mathrm{g}}}{c} - 1 \geq -3 \times 10^{-15}}$$



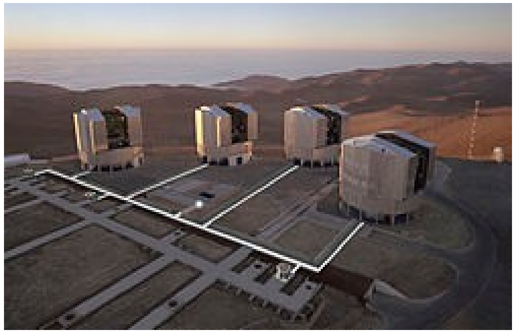
Sgr A* - Central SMBH of the Milky Way





- Stellar orbits around Sgr A* have been monitored since 1992 by 2 groups led by 2020 Nobel Prize winners:
  - R. Genzel (ESO): NTT/VLT, Chile
  - A. Ghez: Keck, Hawaii, USA

New Technology Telescope (3.6 m), La Silla Obs., Chile

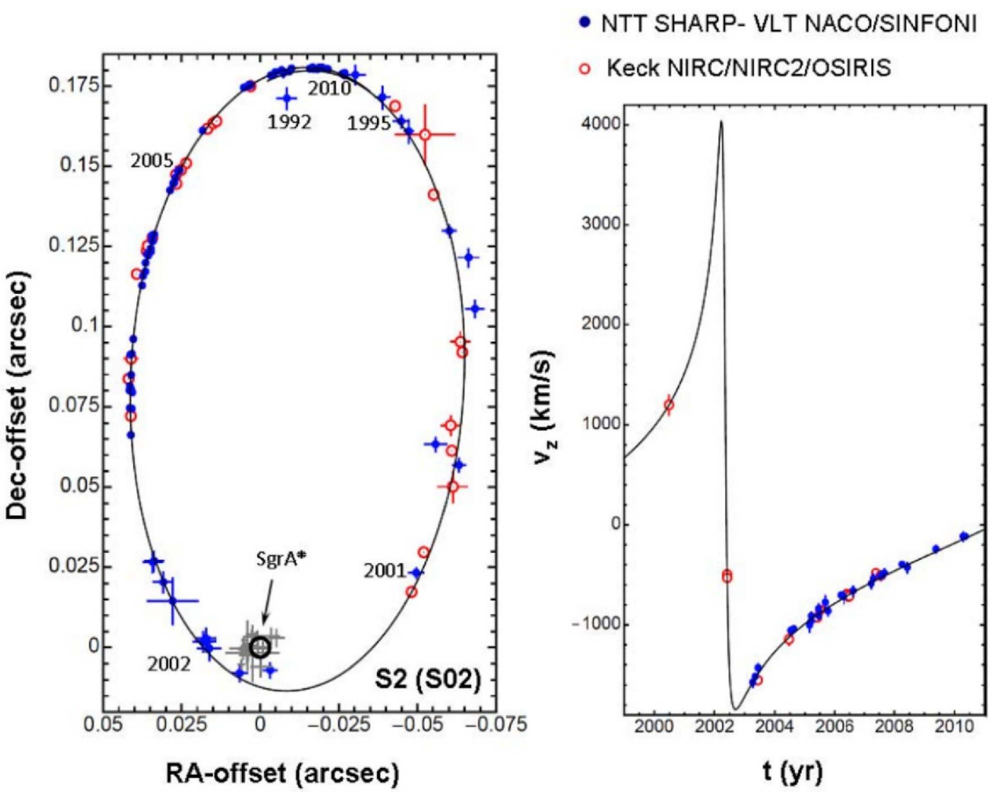


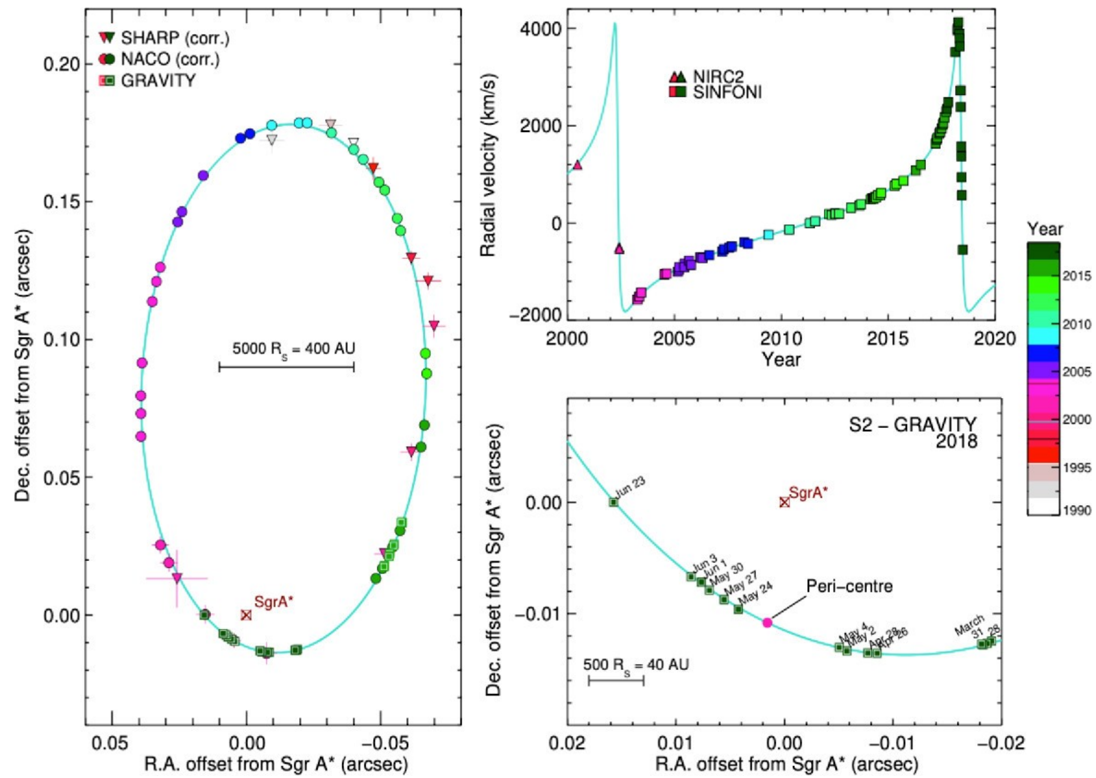
Very Large Telescope (4 x 8.2 m), Paranal Obs., Chile

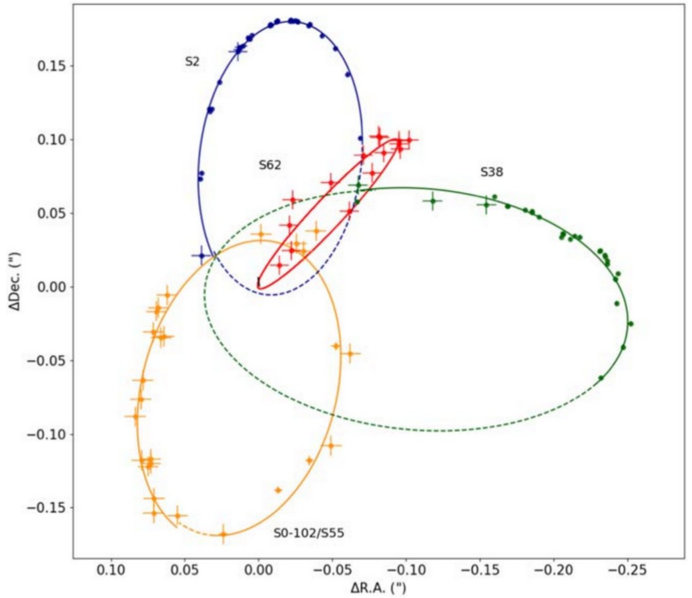
Keck telescope (2 x 10 m), Hawaii, USA

Observed stellar orbits around Sgr A*

- Observations of S2 star by NTT/VLT and Keck (below):
  - Gillessen et al. 2009, ApJ, 692, 1075
  - Ghez et al. 2008, ApJ, 689, 1044
  - Gillessen et al. 2017, ApJ, 837, 30

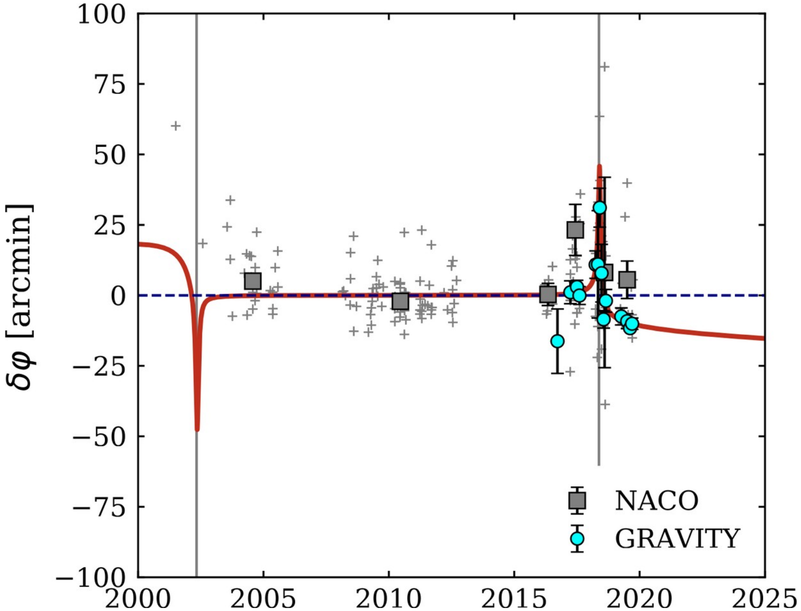


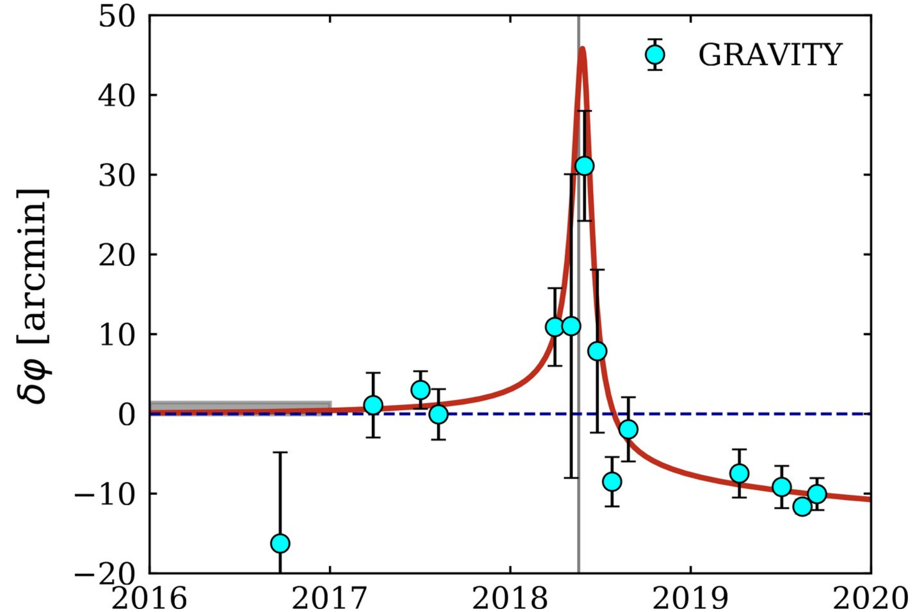




- New observations (right):
  - GRAVITY, 2018, A&A, 615, L15
  - Peißker et al. 2020, ApJ, 889, 61

Detection of Schwarzschild precession in S2 star orbit





The precession angle projected on the sky as a function of time, i.e. the difference between the best-fit GR orbit for $f_{SP} = 1.1$ (thick red curve) and the same orbit for $f_{SP} = 0$ (Newton + Rømer effect + transverse Doppler effect + gravitational redshift)

$$\Delta\phi_{\text{per orbit}} = \text{PPN1}_{\text{SP}} = f_{\text{SP}} \frac{3\pi R_{\text{S}}}{a(1-e^2)} \overset{\text{for S2}}{=} f_{\text{SP}} \times 12.1' \quad (f_{\text{SP}} = 0 \text{ for Newton and } 1 \text{ for GR})$$

- f_{SP} - an ad hoc factor in front of the first post-Newtonian correction of GR showing to which extent some gravitational model is relativistic
- The best-fit value from the observed orbit of S2 star around Sgr A*: $f_{\text{SP}} = 1.10 \pm 0.19$

A&A 636, L5 (2020)
https://doi.org/10.1051/0004-6361/202037813

Astronomy & Astrophysics

Letter to the Editor

Detection of the Schwarzschild precession in the orbit of the star S2 near the Galactic centre massive black hole

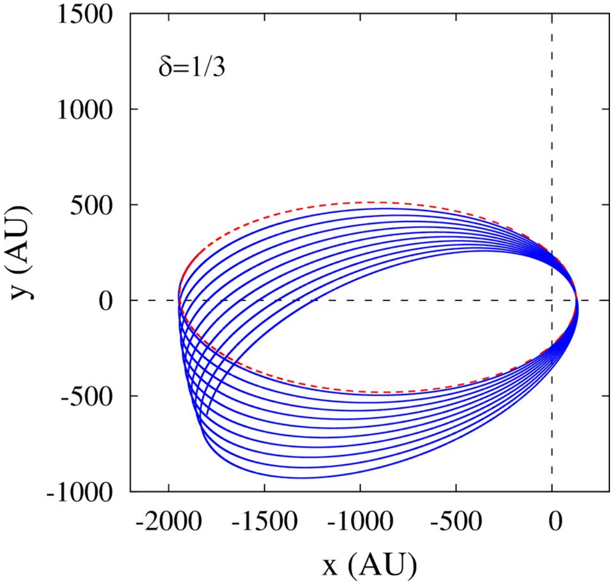
GRAVITY Collaboration:* R. Abuter[8], A. Amorim[6,13], M. Bauböck[1], J. P. Berger[5,8], H. Bonnet[8], W. Brandner[3],

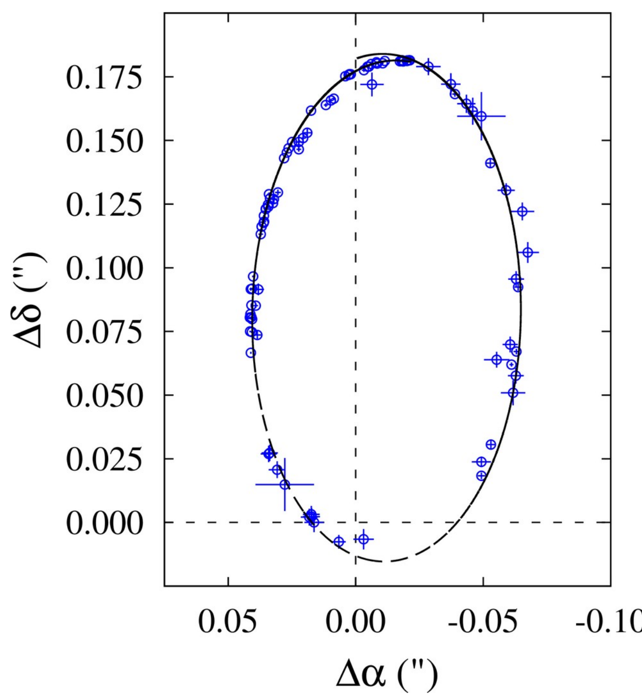
Simulated orbit of S2 star in Yukawa gravity

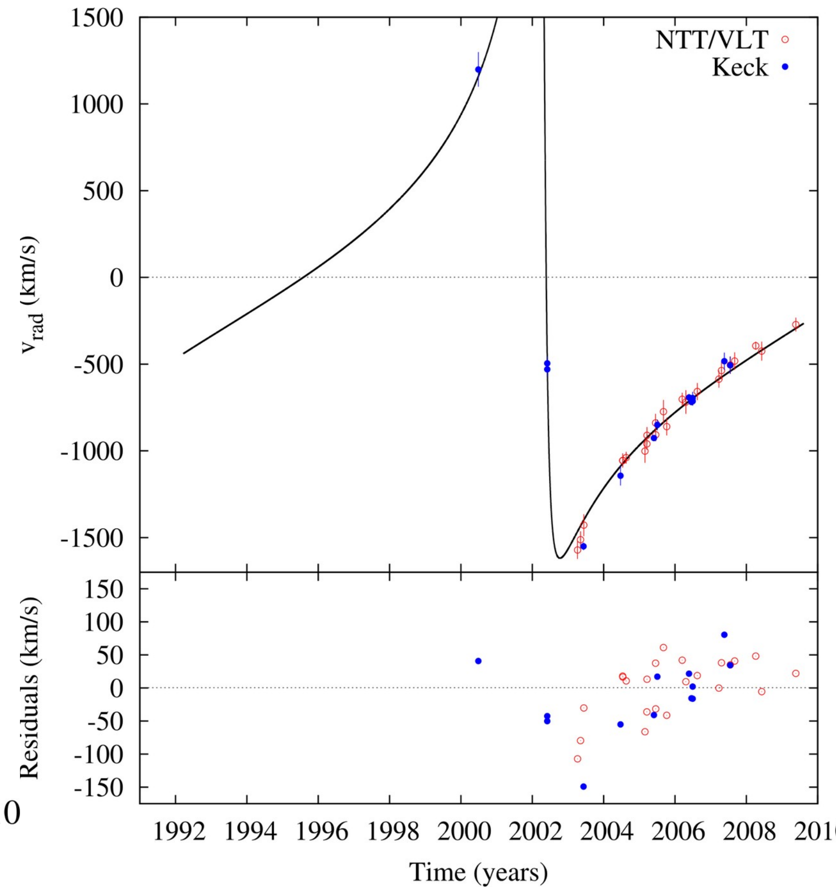
- Numerical integration of differential equations of motion (Borka, Jovanović, Borka Jovanović, Zakharov, 2013, JCAP, 2013, No. 11, 050): $\dot{\vec{r}} = \vec{v}, \ \mu\ddot{\vec{r}} = -\nabla\Phi\left(\vec{r}\right)$
- Equation of motion (EoM):

$$\Phi\left(r\right) = -\frac{GM}{(1+\delta)r}\left(1+\delta e^{-\dfrac{r}{\Lambda}}\right) \Rightarrow \boxed{\ddot{\vec{r}} = -\frac{G(M+m)}{1+\delta}\left[1+\delta\left(1+\frac{r}{\Lambda}\right)e^{-\dfrac{r}{\Lambda}}\right]\frac{\vec{r}}{r^3}}$$

- Simulated orbits were then fitted to the astrometric observations of S2 star

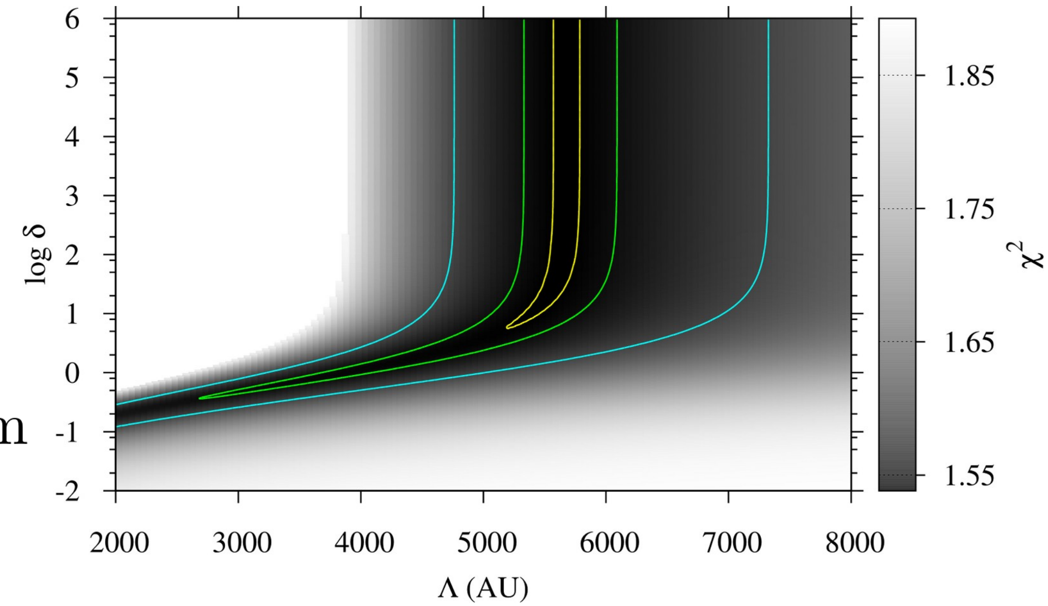


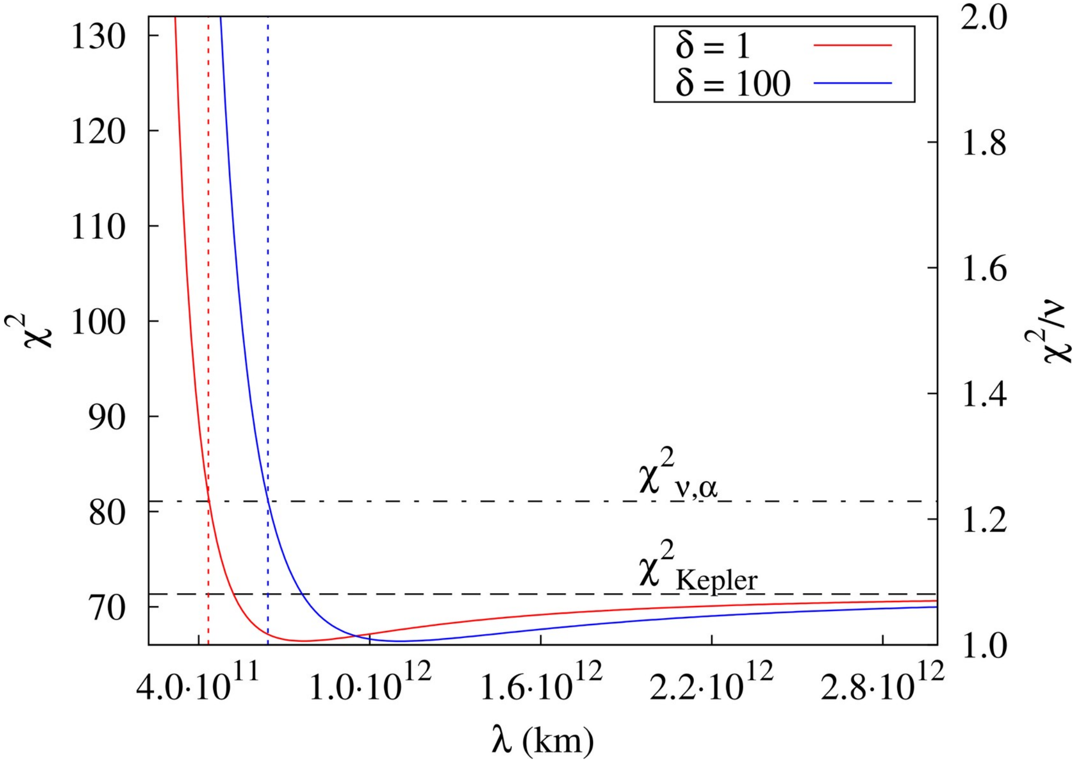




Our estimates for graviton mass upper bound

- χ^2 test of goodness of the fits for significance level $\alpha = 0.1$: regions $\lambda < \lambda_x$ where $\chi^2 > \chi^2_{\nu,\alpha}$ can be excluded with 90% probability
- $\delta = 1: \ \lambda_x \approx 2900\,\mathrm{AU} \approx 4.3 \times 10^{11}$ km
- $\delta = 100: \ \lambda_x \approx 4300\,\mathrm{AU} \approx 6.4 \times 10^{11}$ km
- Corresponding upper bounds for graviton mass: $m_g < h\,c/\lambda_x \ \Rightarrow$
- $\delta = 1:$ $\boxed{m_g < 2.9 \times 10^{-21}\ \mathrm{eV}}$
- $\delta = 100:$ $\boxed{m_g < 1.9 \times 10^{-21}\ \mathrm{eV}}$





Journal of Cosmology and Astroparticle Physics
An IOP and SISSA journal

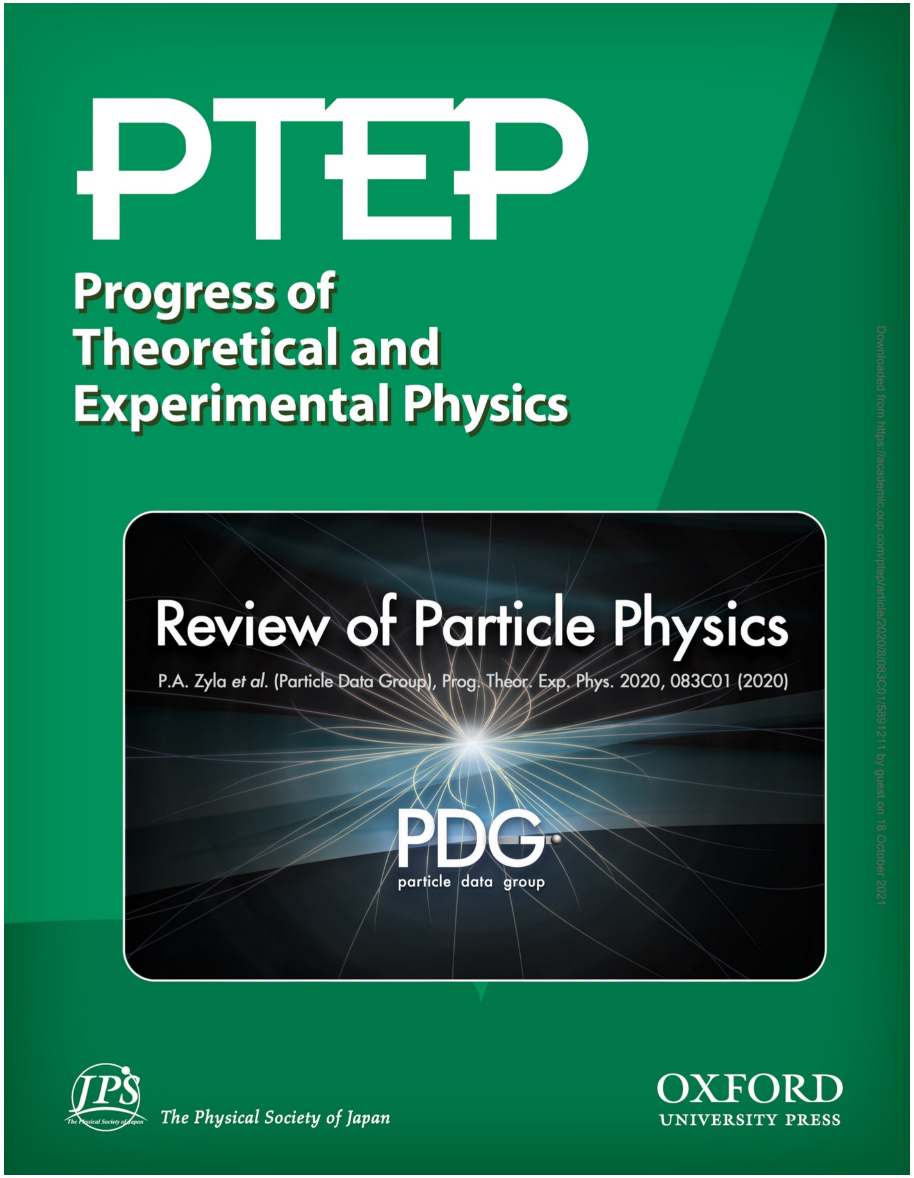
JCAP05(2016)045

Constraining the range of Yukawa gravity interaction from S2 star orbits II: bounds on graviton mass

A.F. Zakharov,[a,b,c,d,e] P. Jovanović,[f] D. Borka[g] and V. Borka Jovanović[g]

Our estimates for graviton mass accepted by PDG

- From 2019, our estimate is in *Gauge and Higgs Boson Particle Listings* by PDG (Zyla et al., PDG, 2020, PTEP, 083C01)



1014

Gauge & Higgs Boson Particle Listings

γ, g, graviton, W

graviton

$J = 2$

graviton MASS

VALUE (eV)	DOCUMENT ID	TECN	COMMENT
$<6 \times 10^{-32}$	[1] CHOUDHURY 04	YUKA	Weak gravitational lensing
• • • We do not use the following data for averages, fits, limits, etc. • • •			
$<6.8 \times 10^{-23}$	BERNUS 19	YUKA	Planetary ephemeris INPOP17b
$<1.4 \times 10^{-29}$	[2] DESAI 18	YUKA	Gal cluster Abell 1689
$<5 \times 10^{-30}$	[3] GUPTA 18	YUKA	SPT-SZ
$<3 \times 10^{-30}$	[3] GUPTA 18	YUKA	Planck all-sky SZ
$<1.3 \times 10^{-29}$	[3] GUPTA 18	YUKA	redMaPPer SDSS-DR8
$<6 \times 10^{-30}$	[4] RANA 18	YUKA	Weak lensing in massive clusters
$<8 \times 10^{-30}$	[5] RANA 18	YUKA	SZ effect in massive clusters
$<7 \times 10^{-23}$	[6] ABBOTT 17	DISP	Combined dispersion limit from three BH mergers
$<1.2 \times 10^{-22}$	[6] ABBOTT 16	DISP	Combined dispersion limit from two BH mergers
$<2.9 \times 10^{-21}$	[7] ZAKHAROV 16	YUKA	S2 star orbit
$<5 \times 10^{-23}$	[8] BRITO 13		Spinning black holes bounds
$<4 \times 10^{-25}$	[9] BASKARAN 08		Graviton phase velocity fluctuations
$<6 \times 10^{-32}$	[10] GRUZINOV 05	YUKA	Solar System observations
$<9.0 \times 10^{-34}$	[11] GERSHTEIN 04		From Ω_{tot} value assuming RTG
$>6 \times 10^{-34}$	[12] DVALI 03		Horizon scales
$<8 \times 10^{-20}$	[13,14] FINN 02	DISP	Binary pulsar orbital period decrease
	[14,15] DAMOUR 91		Binary pulsar PSR 1913+16
$<7 \times 10^{-23}$	TALMADGE 88	YUKA	Solar system planetary astrometric data
$< 2 \times 10^{-29}\ h_0^{-1}$	GOLDHABER 74		Rich clusters
$<7 \times 10^{-28}$	HARE 73		Galaxy
$<8 \times 10^{4}$	HARE 73		2γ decay

graviton REFERENCES

ABBOTT 16 PRL 116 061102 B.P. Abbott *et al.* (LIGO and Virgo Collabs.)
ZAKHAROV 16 JCAP 1605 045 A.F. Zakharov *et al.*
BRITO 13 PR D88 023514 R. Brito, V. Cardoso, P. Pani (LISB, MISS, HSCA+)

Impact of our studies of fifth force

PRL **118,** 211101 (2017) PHYSICAL REVIEW LETTERS week ending 26 MAY 2017

Testing General Relativity with Stellar Orbits around the Supermassive Black Hole in Our Galactic Center

A. Hees,[1*] T. Do,[1] A. M. Ghez,[1†] G. D. Martinez,[1] S. Naoz,[1] E. E. Becklin,[1] A. Boehle,[1] S. Chappell,[1] D. Chu,[1] A. Dehghanfar,[1] K. Kosmo,[1] J. R. Lu,[2] K. Matthews,[3] M. R. Morris,[1] S. Sakai,[1] R. Schödel,[4] and G. Witzel[1]

- Our estimates for graviton mass are recognized by 2 groups monitoring the stellar orbits around Sgr A*

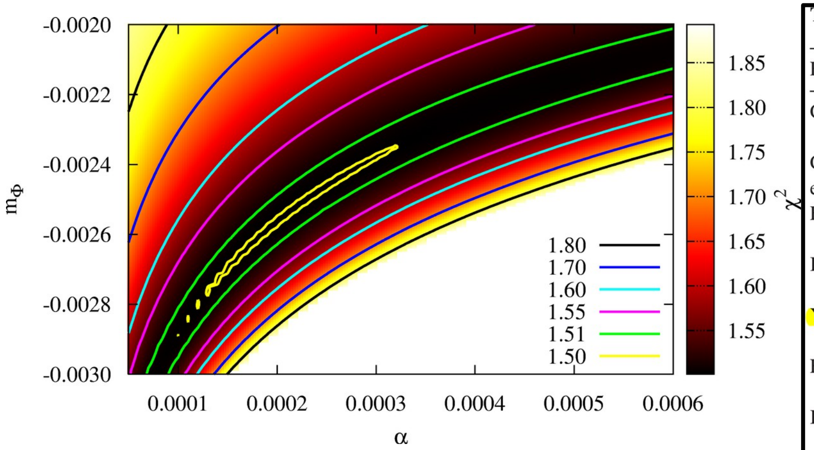
A specific theoretical model covered by the fifth force framework is a massive graviton. In that context, we found a 90% confidence limit $\lambda > 5000$ A.U. for $\alpha = 1$, which can be interpreted as a lower limit on the graviton's Compton wavelength $\lambda_g > 7.5 \times 10^{11}$ km or, equivalently, as an upper bound on the graviton's mass $m_g < 1.6 \times 10^{-21}$ eV$/c^2$ (see also Ref. [36]). This constraint is one order of magnitude less stringent than the recent bound obtained by LIGO [78], which, nevertheless, does not apply for all models predicting a fifth force.

[36] D. Borka, P. Jovanović, V. B. Jovanović, and A. F. Zakharov, Constraining the range of Yukawa gravity interaction from S2 star orbitsJ. Cosmol. Astropart. Phys. 11 (2013) 050; A. F. Zakharov, P. Jovanović, D. Borka, and V. B. Jovanović, Constraining the range of Yukawa gravity interaction from S2 star orbits II: Bounds on graviton mass, J. Cosmol. Astropart. Phys. 05 (2016) 045.

Our tests of several modified gravity theories

- Sanders-like potential (Capozziello, Borka, Jovanović, Borka Jovanović, 2014, PRD, 90, 044052):

$$\Phi(\mathbf{x}) = -\frac{G_\infty M}{|\mathbf{x}|}\left\{1+\alpha\, e^{-\sqrt{1-3\alpha}\, m_\phi |\mathbf{x}|}\right\}$$



MNRAS 489, 4606–4621 (2019) doi:10.1093/mnras/stz2300
Advance Access publication 2019 August 22

Scalar field effects on the orbit of S2 star

The GRAVITY Collaboration : A. Amorim,[1,2] M. Bauböck,[3] M. Benisty,[4] J.-P. Berger,[4] Y. Clénet,[5] V. Coudé du Forest,[5] T. de Zeeuw,[3,6] J. Dexter,[3] G. Duvert,[4] A. Eckart,[7,8] F. Eisenhauer,[3] Miguel C. Ferreira,[1★] F. Gao,[3] Paulo J. V. Garcia,[1,9,10★] E. Gendron,[5] R. Genzel,[3,11] S. Gillessen,[3]

Table 2. Literature computing extensions/alternatives to GR effects in the orbits of the S-stars.

Extension/alternative	Results/comments	Reference
Charged non-rotating black holes	Upper limit to black hole charge from S2 precession upper limit.	De Laurentis et al. (2018a), Iorio (2012), Zakharov (2018)
Charged rotating black holes and plasma effects	Upper limits from black hole mass, spin, and local magnetic field	Zajaček et al. (2018)
Fermion ball	Ruled out by Ghez et al. (2005) and Gravity Collaboration (2018a).	Munyaneza & Viollier (2002)
Boson 'star'	Effects much smaller than GR at S2 orbit, only relevant at a few tens of Schwarzschild radii.	Amaro-Seoane et al. (2010), Boshkayev & Malafarina (2019), Grould et al. (2017a)
Yukawa potential	Upper limits on potential parameters and graviton mass from S2 precession upper limit.	Borka et al. (2013), Hees et al. (2017), Zakharov et al. (2016), Zakharov et al. (2018)
Einstein–Maxwell–Dilaton–Axion gravity	Effects smaller than 10^{-3} of GR for S2, need pulsars or inner stars for further tests.	De Laurentis et al. (2018a)
Brans–Dicke theory	Effects smaller than 10^{-3} of GR for S2, need pulsars or inner stars for further tests.	De Laurentis et al. (2018a), Kalita (2018)
f(*R*) gravity	Effects smaller than 10^{-3} of GR for S2, need pulsars or inner stars for further tests.	Capozziello et al. (2014), De Laurentis et al. (2018a), De Laurentis, De Martino & Lazkoz (2018b), Kalita (2018)
Non-local gravity	Precession compatible with observational upper limit, of the order of GR prediction	Dialektopoulos et al. (2019)
Scalar tensor gravity	Precession is 13× GR value, ruled out by Hees et al. (2017)	Borka Jovanović et al. (2019)
$f(R, \phi)$ gravity	Best-fitting precession prediction for S2 is 20× GR value, ruled out by Hees et al. (2017)	Capozziello et al. (2014)
Hybrid gravity	Best-fitting precession prediction too high, ruled out by Hees et al. (2017)	Borka et al. (2016)
R^n gravity	When compared with Hees et al. (2017) upper value, the GR value ($n = 1$) is recovered to <1 per cent, or smaller if extended mass distributions are present	Borka et al. (2012), Zakharov et al. (2014)
Quadratic Einstein–Gauss–Bonnet gravity	Derive expressions for gravitational redshift in function of theory coupling parameters (scalar/matter and scalar/Gauss–Bonnet invariant).	Hees et al. (2019)
Dark matter profiles (See Table 1 for dark matter + black hole studies)	Dark matter mass required to explain TeV emission compatible with orbital upper limits. Limits on spatial distribution of non-annihilating dark matter.	de Paolis et al. (2011), Dokuchaev & Eroshenko (2015), Hall & Gondolo (2006), Iorio (2013), Lacroix (2018), Zakharov et al. (2007)
Scalar fields and ultralight dark matter	Upper limits on scalar field mass (1 pe rcent of black hole) for particles of mass 4×10^{-19} eV c^{-2}	Bar et al. (2019)

809, 127 (2015); D. Borka, P. Jovanović, V. B. Jovanović, and A. F. Zakharov, Constraints on R^n gravity from precession of orbits of S2-like stars, Phys. Rev. D **85**, 124004 (2012); S. Capozziello, D. Borka, P. Jovanović, and V. B. Jovanović, Constraining extended gravity models by S2 star orbits around the Galactic centre, Phys. Rev. D **90**, 044052 (2014); A. F. Zakharov, D. Borka, V. B. Jovanović, and P. Jovanović, Constraints on R^n gravity from precession of orbits of S2-like stars: A case of a bulk distribution of mass, Adv. Space Res. **54**, 1108 (2014); D. Borka, S. Capozziello, P. Jovanović, and V. B. Jovanović, Probing hybrid modified gravity by stellar motion around Galactic centre, Astropart. Phys. **79**, 41 (2016).

Parametrized Post-Newtonian (PPN) formalism

- PPN formalism completely characterizes the weak-field behavior of a gravity theory by a set of 10 PPN parameters in which this theory differs from Newtonian gravity
- Expansion in terms of 10 dimensionless metric potentials (Will, 2014, LRR, 17, 4):

$$U, U_{ij}, \Phi_W, A, \Phi_1, \Phi_2, \Phi_3, \Phi_4, V_i, W_i$$

- Metric:

$$g_{00} = -1 + 2U - 2\beta U^2 - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 + \ldots + \mathcal{O}(\epsilon^3)$$

$$g_{0i} = -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i - \ldots + \mathcal{O}(\epsilon^{5/2})$$

$$g_{ij} = (1 + 2\gamma U)\delta_{ij} + \mathcal{O}(\epsilon^2)$$

- Precession angle per orbit:

$$\Delta\phi \approx (2 + 2\gamma - \beta)\,\frac{2\pi GM}{c^2 a\,(1 - e^2)}$$

- Light deflection angle: $\alpha \approx 2\,(1+\gamma)\,\dfrac{GM}{c^2\xi}$
- Equation of motion:

$$\ddot{\vec{r}} = -GM\frac{\vec{r}}{r^3} + \frac{GM}{c^2 r^3}\left\{\left[2\,(\beta+\gamma)\,\frac{GM}{r} - \gamma\left(\dot{\vec{r}}\cdot\dot{\vec{r}}\right)\right]\vec{r} + 2\,(1+\gamma)\left(\vec{r}\cdot\dot{\vec{r}}\right)\dot{\vec{r}}\right\}$$

Parameter	What it measures relative to GR	Value in GR
γ	How much space-curvature produced by unit rest mass?	1
β	How much "nonlinearity" in the superposition law for gravity?	1
ξ	Preferred-location effects?	0
α_1	Preferred-frame effects?	0
α_2		0
α_3		0
α_3	Violation of conservation of total momentum?	0
ζ_1		0
ζ_2		0
ζ_3		0
ζ_4		0

Extended/modified PPN formalisms

- The standard PPN formalism is not viable for massive gravity theories because Newtonian order terms are modified by the presence of massive fields so that the Newtonian potential acquires a Yukawa-like correction (Clifton, 2008, PRD, 77, 024041; Alsing et al. 2012, PRD, 85, 064041)

- $f(R)$ gravity in the low energy limit gives the Yukawa potential, but it also includes the first post-Newtonian approximation that in the limit $f(R) \rightarrow R$ should coincide with GR $\Rightarrow$ **extended PPN formalism** which includes both Yukawa correction and the post-Newtonian correction (Jovanović et al. 2023, JCAP, 056):

$$\ddot{\vec{r}}_N = -GM\frac{\vec{r}}{r^3}, \qquad \ddot{\vec{r}}_{cor,GR} = \frac{GM}{c^2 r^3}\left[\left(4\frac{GM}{r} - \dot{\vec{r}}\cdot\dot{\vec{r}}\right)\vec{r} + 4\left(\vec{r}\cdot\dot{\vec{r}}\right)\dot{\vec{r}}\right],$$

$$\ddot{\vec{r}}_{cor,Y} = \frac{\delta \cdot GM}{1+\delta}\left[1 - \left(1 - \frac{r}{\Lambda}\right)e^{-\frac{r}{\Lambda}}\right]\frac{\vec{r}}{r^3}$$

1. Standard PPN EoM in GR: $\boxed{\ddot{\vec{r}}_{GR} = \ddot{\vec{r}}_N + \ddot{\vec{r}}_{cor,GR}}$
2. Extended PPN EoM in Yukawa gravity: $\boxed{\ddot{\vec{r}}_Y = \ddot{\vec{r}}_N + \ddot{\vec{r}}_{cor,GR} + \ddot{\vec{r}}_{cor,Y}}$
3. Modified PPN EoM which was used by GRAVITY Collaboration to parametrize the effect of the Schwarzschild metric: $\boxed{\ddot{\vec{r}}_{SP} = \ddot{\vec{r}}_N + f_{SP}\,\ddot{\vec{r}}_{cor,GR}}$

Simulating S2 star orbit using PPN EoMs

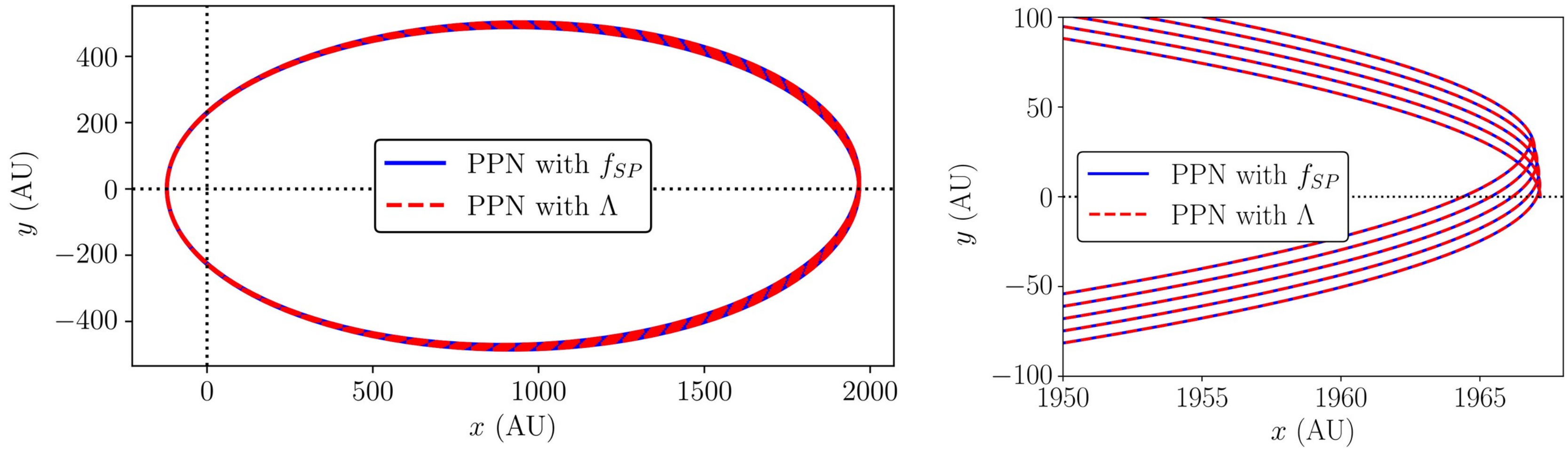


FIG. 1. Comparisons between the simulated orbits of S2 star, obtained by numerical integration of equation of motions in PPN formalism (2) for $f_{SP} = 1.10$ (blue solid line) and in PPN formalism (3) for $\Lambda = 46924.6$ AU (red dashed line). The orbits are calculated during five orbital periods, and their zoomed parts around the apocenter, where the largest discrepancy could occur, are presented in the right panel for better insight.

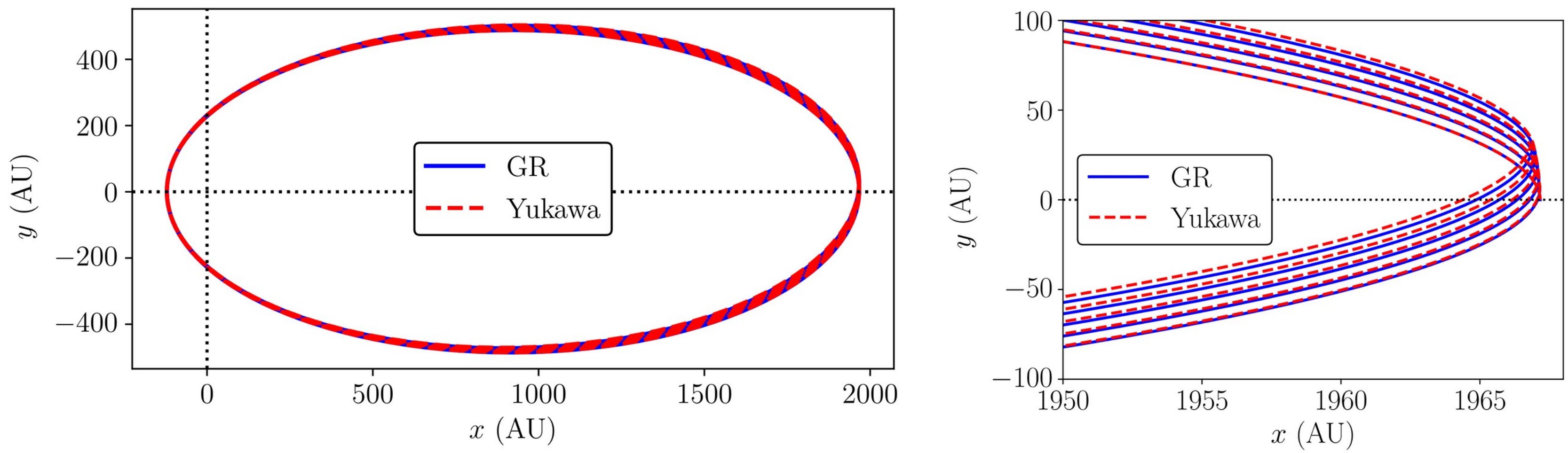
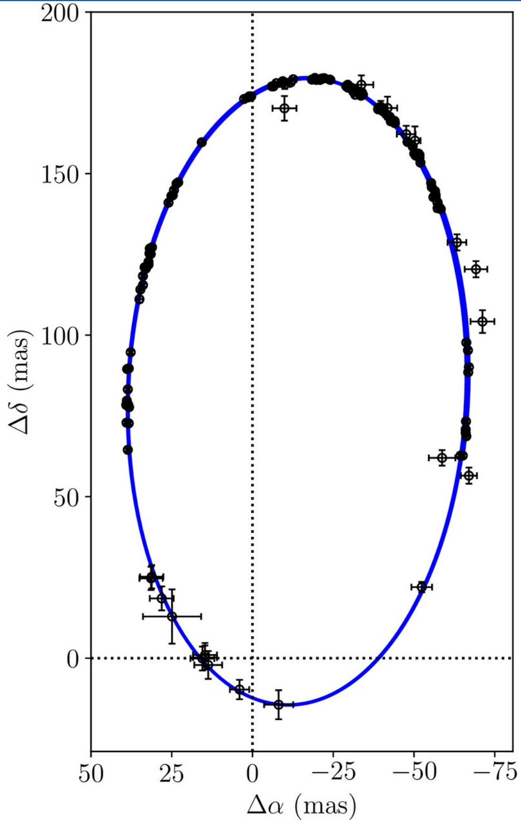
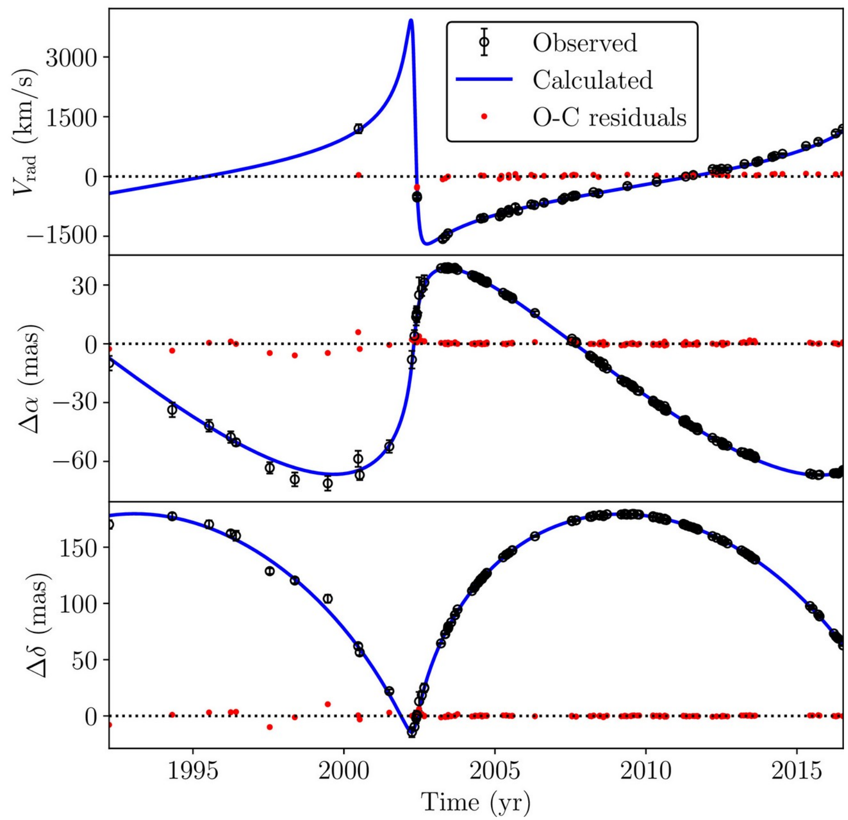


FIG. 2. Comparisons between the simulated orbits of S2 star, obtained by numerical integration of PPN equation in GR (blue solid line) and in Yukawa gravity for $\Lambda = 46924.6$ AU (red dashed line, which corresponds to $f_{SP} = 1.10$) using PPN formalism (3). The orbits are calculated during five orbital periods, and their zoomed parts around the apocenter, where the largest discrepancy could occur, are presented in the right panel for better insight.

Direct fitting of S2 star orbit





Parameter	Value	Fit Error	Unit
λ	82,175.7	9828.05	AU
M	4.15	0.27	$10^6\ M_\odot$
R	8.33	0.198	kpc
a	0.1229	0.00430	arcsec
e	0.8797	0.01597	
i	134.89	1.984	∘
Ω	224.57	5.208	∘
ω	62.78	4.562	∘
P	15.98	0.362	yr
T	2018.12219	0.696709	yr

- Graviton mass upper bound from the best fit orbit of S2 star in phenomenological potential by C. Will: $m_g < (1.5 \pm 0.8) \times 10^{-22}$ eV

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Article

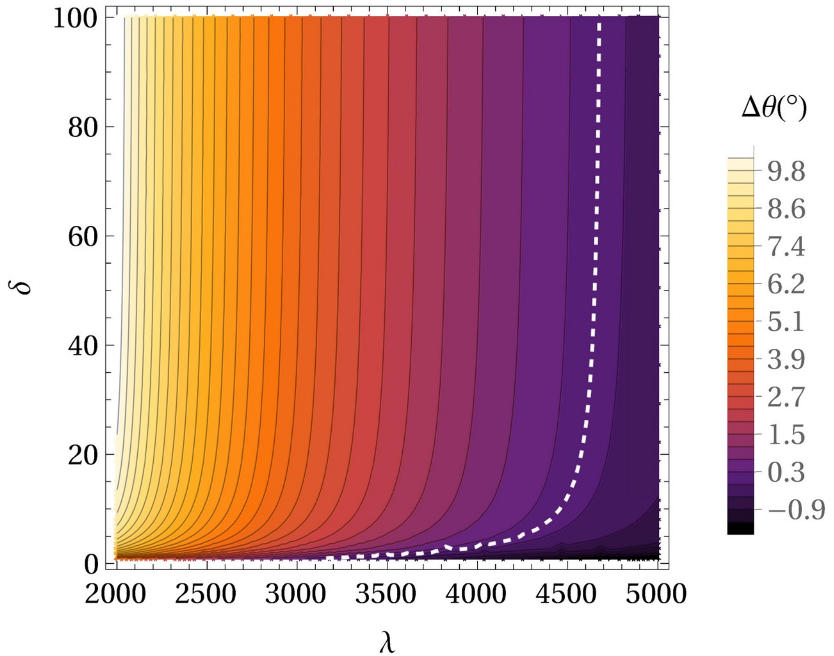
Constraints on Graviton Mass from Schwarzschild Precession in the Orbits of S-Stars around the Galactic Center

Predrag Jovanović [1,*], Vesna Borka Jovanović [2], Duško Borka [2] and Alexander F. Zakharov [3]

Influence of bulk distribution of matter

- Orbital precession of S2 star in Yukawa gravity for different mass densities of bulk distribution of matter which describes stellar cluster, interstellar gas and dark matter, contained within some radius r around SMBH: $M(r) = M_{BH} + M_{ext}(r)$
- Double power-law mass density profile (where $r_0 = 10'' \wedge \alpha = 1.4$ for S2 star):

$$\rho(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-\alpha}, \ \alpha = \begin{cases} 2.0 \pm 0.1, & r \geq r_0 \\ 1.4 \pm 0.1, & r < r_0 \end{cases} \Rightarrow M_{ext}(r) = \frac{4\pi\rho_0 r_0^{\alpha}}{3-\alpha} r^{3-\alpha}$$



- S2 star precession per orbital period in (λ, δ) parameter space for mass density of extended matter: $\rho_0 = 2 \times 10^8\ M_{\odot}\mathrm{pc}^{-3}$
- White dashed line: the case when the precession is the same as in GR $(0^{\circ}.18)$

Table 1 The values of parameter λ (in AU) for different combinations of 3 values of parameter δ the 5 values of the mass density distribution of extended matter ρ_0

	ρ_0 (in $10^8 M_{\odot}\mathrm{pc}^{-3}$)				
	0	2	4	6	8
δ=1	15125	3130	2080	1597	1302
δ=10	20395	4425	3015	2370	1978
δ=100	21285	4640	3175	2500	2090

Table 2 The graviton mass (m_g) estimates corresponding to all mass density distributions presented in Table 1, in the case when Yukawa gravity parameter $\delta = 1$

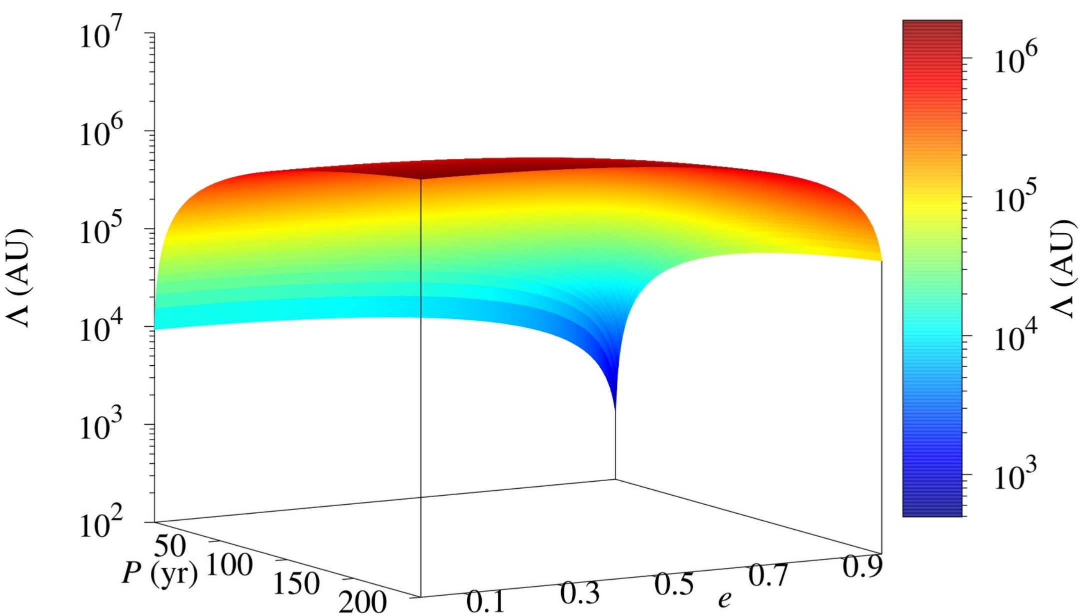
ρ_0 (in $10^8 M_{\odot}\mathrm{pc}^{-3}$)	0	2	4	6	8
m_g (in 10^{-21} eV)	0.5	2.6	4.0	5.2	6.4

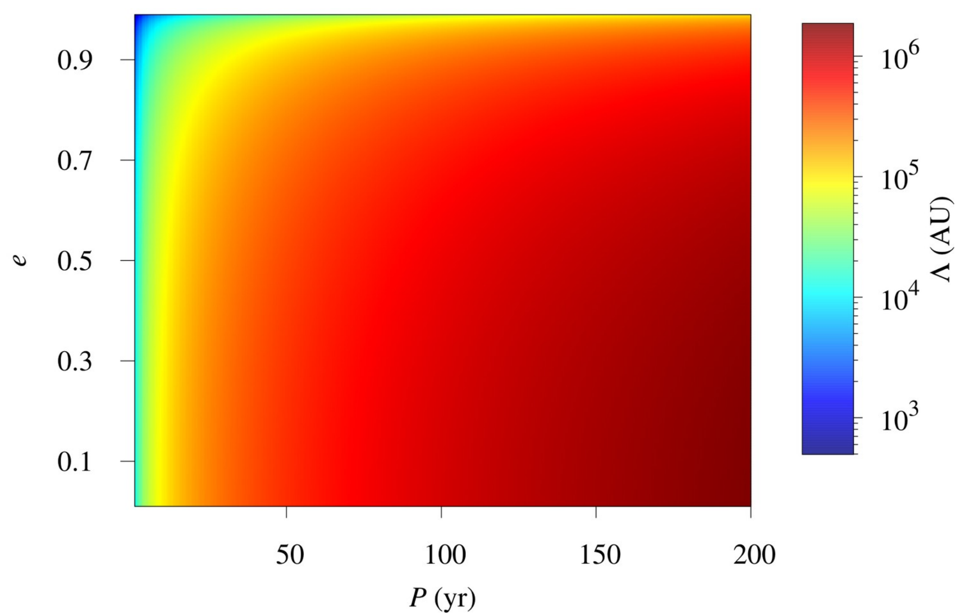
(Jovanović, Borka, Borka Jovanović, Zakharov, 2021, EPJD, 75, 145)

Constraints from detected Schwarzschild precession

- Schwarzschild precession: $\Delta\varphi_{GR}^{rad} \approx \dfrac{6\pi GM}{c^2 a(1-e^2)}$
- Additional contribution of Yukawa gravity: $\Delta\varphi_{Y}^{rad} \approx \dfrac{\pi\delta\sqrt{1-e^2}}{1+\delta}\dfrac{a^2}{\Lambda^2}, \qquad a \ll \Lambda$
- Observed precession detected by GRAVITY: $\Delta\varphi_{obs} \approx \dfrac{2\pi GM}{c^2 a(1-e^2)}(3f_{sp})$
- Constraints on the range of Yukawa interaction Λ from observed S-star orbits:

$$\Delta\varphi_Y + \Delta\varphi_{GR} \approx \Delta\varphi_{obs} \quad \Rightarrow \quad \boxed{\Lambda(P,e;\delta) \approx \frac{cP}{2\pi}\sqrt{\frac{\delta(\sqrt{1-e^2})^3}{2(3f_{sp}-3)(1+\delta)}}}$$





Dependence of range of a fifth force on its strength

- Results for all S-stars from Table 3 in Gillessen et al 2017, ApJ, 837, 30, except of S111
- A fifth force with greater strength δ has a greater range Λ
- In case $\delta \gtrsim 10$ there is very little difference compared to case $\delta = 1$

Star	$\Delta\varphi$ (")	P^* (yr)	$\Lambda \pm \Delta\Lambda$ (AU) $\delta = 0.01$	$\delta = 0.1$	$\delta = 1$
S1	48.2	125.79	164626.9 ± 13537.8	498844.4 ± 41021.7	1169893.8 ± 96204.5
S2	722.1	5.12	6730.8 ± 149.9	20395.2 ± 454.3	47831.0 ± 1065.4
S4	65.5	68.01	89185.0 ± 1750.2	270244.0 ± 5303.3	633778.4 ± 12437.2
S6	102.4	76.74	100839.1 ± 267.5	305557.6 ± 810.6	716596.2 ± 1901.1
S8	138.0	42.73	56050.5 ± 1717.2	169841.6 ± 5203.5	398314.0 ± 12203.2
S9	124.3	34.33	45030.4 ± 2503.1	136448.8 ± 7584.9	320000.8 ± 17788.2
S12	314.6	18.33	24050.7 ± 475.7	72877.1 ± 1441.4	170912.0 ± 3380.4
S13	91.6	42.20	55328.1 ± 601.8	167652.4 ± 1823.5	393179.8 ± 4276.6
S14	1465.9	5.60	7346.3 ± 981.2	22260.4 ± 2973.2	52205.2 ± 6972.8
S17	66.1	67.35	88370.7 ± 4262.7	267776.5 ± 12916.7	627991.6 ± 30292.4
S18	107.1	34.71	45506.5 ± 926.2	137891.7 ± 2806.5	323384.7 ± 6581.8
S19	87.1	72.62	95481.5 ± 36447.7	289323.6 ± 110442.1	678523.9 ± 259009.7
S21	217.4	19.18	25142.3 ± 1261.7	76185.0 ± 3823.2	178669.6 ± 8966.2
S22	19.0	456.10	599449.8 ± 236689.3	1816423.7 ± 717204.5	4259891.2 ± 1681993.5
S23	114.1	34.54	45425.0 ± 11014.4	137644.5 ± 33375.3	322805.0 ± 78272.1
S24	107.5	97.28	127590.2 ± 14036.6	386617.6 ± 42533.2	906698.7 ± 99749.1
S29	98.5	57.33	75236.9 ± 14099.5	227979.3 ± 42723.7	534658.8 ± 100196.0
S31	63.3	82.46	108733.3 ± 3953.7	329478.4 ± 11980.3	772695.3 ± 28096.4
S33	47.9	135.83	178323.3 ± 27097.8	540346.5 ± 82110.5	1267224.9 ± 192566.2
S38	427.5	8.31	10917.4 ± 51.8	33081.5 ± 157.1	77582.9 ± 368.4
S39	364.5	19.25	25286.5 ± 2038.3	76622.1 ± 6176.3	179694.7 ± 14484.7
S42	30.8	250.45	327668.0 ± 127216.9	992883.5 ± 385486.4	2328518.3 ± 904045.8
S54	81.5	144.02	187868.3 ± 301214.1	569269.5 ± 912724.3	1335055.3 ± 2140528.3
S55	382.8	7.38	9666.0 ± 302.1	29289.4 ± 915.3	68689.7 ± 2146.6
S60	105.5	50.59	66370.5 ± 2549.6	201112.8 ± 7725.8	471651.3 ± 18118.6
S66	13.4	655.82	860607.2 ± 88872.3	2607770.2 ± 269296.7	6115763.2 ± 631556.7
S67	19.3	402.94	528751.9 ± 32803.8	1602198.3 ± 99400.4	3757488.1 ± 233114.5
S71	106.2	100.28	131669.4 ± 20154.0	398978.3 ± 61069.7	935687.0 ± 143221.2
S83	15.3	589.30	773374.9 ± 184565.2	2343443.1 ± 559260.5	5495861.3 ± 1311582.1
S85	11.0	1772.21	2311796.7 ± 3523752.4	7005094.4 ± 10677503.5	16428402.6 ± 25040965.4
S87	7.6	1577.89	2065566.0 ± 200654.2	6258978.1 ± 608012.7	14678604.7 ± 1425916.1
S89	31.0	273.90	358908.7 ± 49485.2	1087547.9 ± 149947.8	2550525.9 ± 351658.7
S91	11.4	891.25	1168818.2 ± 101284.2	3541696.1 ± 306906.4	8306013.7 ± 719759.3
S96	13.6	646.91	848925.2 ± 53447.7	2572372.1 ± 161954.7	6032747.3 ± 379817.5
S97	9.7	1151.43	1516520.2 ± 550839.2	4595286.1 ± 1669126.3	10776901.1 ± 3914448.1
S145	23.6	343.33	452172.2 ± 222048.9	1370150.3 ± 672841.7	3213287.3 ± 1577953.6
S175	1812.0	6.30	8263.6 ± 2000.9	25040.0 ± 6062.9	58723.9 ± 14218.7
R34	18.6	589.73	775088.0 ± 220322.6	2348634.1 ± 667611.0	5508035.2 ± 1565686.6
R44	5.5	2579.33	3444472.6 ± 2260986.2	10437273.8 ± 6851130.7	24477576.8 ± 16067325.7

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Constraints on Yukawa gravity parameters from observations of bright stars

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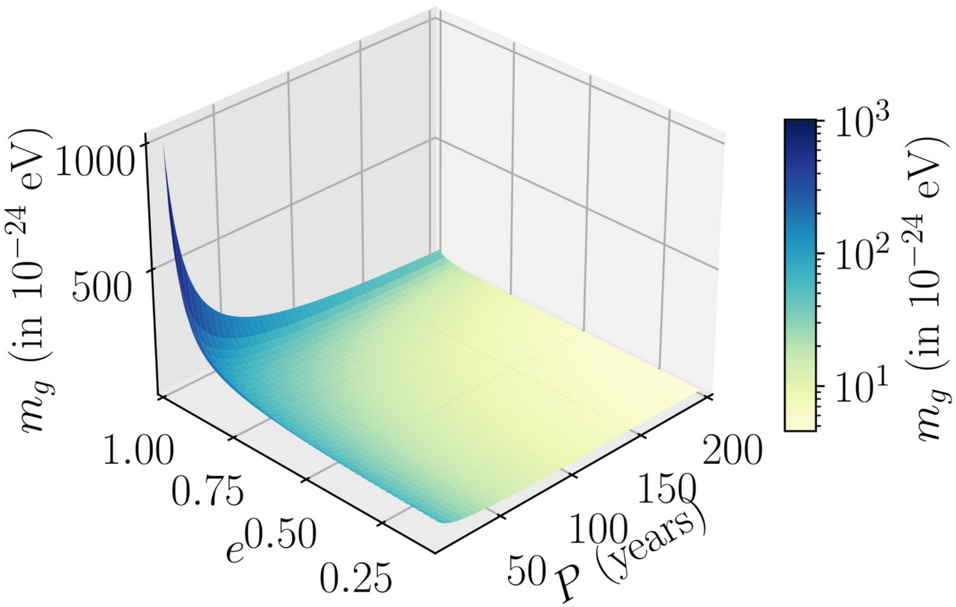
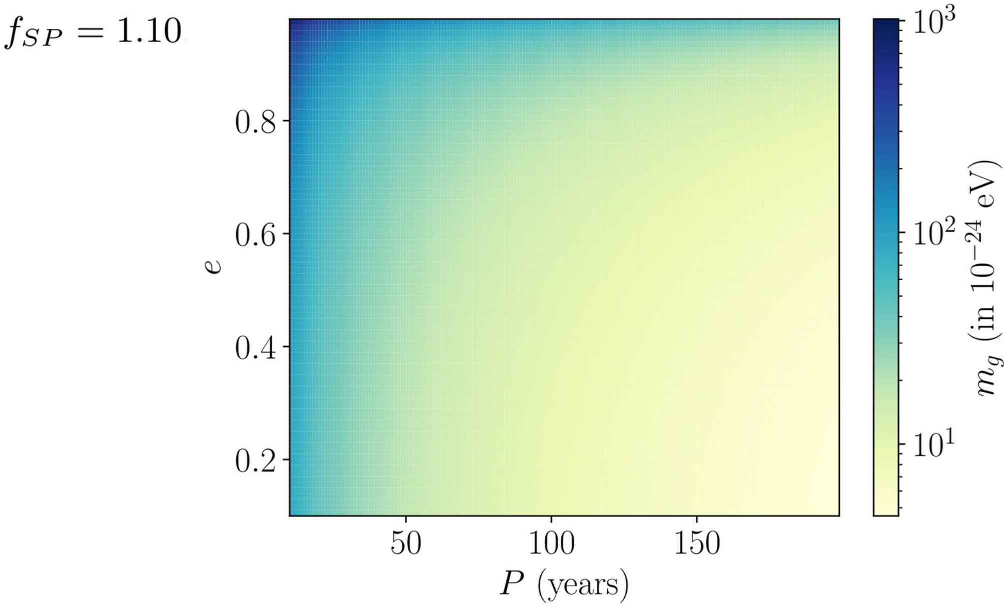
Dependence of graviton mass on f_{SP}

- Assuming that $\delta = 1$ and that Λ corresponds to the graviton Compton wavelength λ_g:

$$\lambda_g = \frac{hc}{m_g} \quad \Rightarrow \quad \boxed{m_g(P, e; f_{SP}) \approx \frac{4\pi h}{P}\sqrt{\frac{3(f_{SP}-1)}{(\sqrt{1-e^2})^3}}}$$





PHYSICAL REVIEW D **109,** 064046 (2024)

Improvement of graviton mass constraints using GRAVITY's detection of Schwarzschild precession in the orbit of S2 star around the Galactic Center

Predrag Jovanović,[1] Vesna Borka Jovanović,[2] Duško Borka,[2,*] and Alexander F. Zakharov[3]

Bounds on range of a fifth force and graviton mass

- Results for all S-stars from Table 3 in Gillessen et al 2017, ApJ, 837, 30, except of S111

Star	$f_{SP} = 1.10 \pm 0.19$			$f_{SP} = 1.19 \pm 0.19$		
	$\Lambda \pm \Delta\Lambda$ (AU)	$m_g \pm \Delta m_g$ (10^{-24} eV)	R.E. (%)	$\Lambda \pm \Delta\Lambda$ (AU)	$m_g \pm \Delta m_g$ (10^{-24} eV)	R.E. (%)
S1	1.2e + 06 ± 1.2e + 06	7.2 ± 7.2	100.7	8.4e + 05 ± 4.7e + 05	9.9 ± 5.5	55.7
S2	4.7e + 04 ± 4.5e + 04	176.6 ± 170.0	96.3	3.4e + 04 ± 1.7e + 04	243.5 ± 124.8	51.3
S4	6.2e + 05 ± 6.0e + 05	13.3 ± 12.9	96.7	4.5e + 05 ± 2.3e + 05	18.3 ± 9.5	51.7
S6	7.0e + 05 ± 6.7e + 05	11.8 ± 11.2	95.2	5.1e + 05 ± 2.6e + 05	16.2 ± 8.2	50.2
S8	3.9e + 05 ± 3.8e + 05	21.2 ± 20.7	98.0	2.8e + 05 ± 1.5e + 05	29.2 ± 15.4	53.0
S9	3.1e + 05 ± 3.1e + 05	26.3 ± 26.2	99.7	2.3e + 05 ± 1.2e + 05	36.3 ± 19.8	54.7
S12	1.7e + 05 ± 1.6e + 05	49.3 ± 47.6	96.4	1.2e + 05 ± 6.3e + 04	68.0 ± 35.0	51.4
S13	3.9e + 05 ± 3.7e + 05	21.4 ± 20.4	95.5	2.8e + 05 ± 1.4e + 05	29.5 ± 14.9	50.5
S14	5.1e + 04 ± 5.5e + 04	161.3 ± 173.2	107.3	3.7e + 04 ± 2.3e + 04	222.4 ± 138.6	62.3
S17	6.2e + 05 ± 6.0e + 05	13.4 ± 13.0	97.1	4.5e + 05 ± 2.3e + 05	18.5 ± 9.6	52.1
S18	3.2e + 05 ± 3.1e + 05	26.0 ± 25.1	96.5	2.3e + 05 ± 1.2e + 05	35.9 ± 18.5	51.5
S19	6.7e + 05 ± 7.8e + 05	12.4 ± 14.5	116.4	4.8e + 05 ± 3.5e + 05	17.2 ± 12.3	71.4
S21	1.8e + 05 ± 1.8e + 05	47.1 ± 46.9	99.6	1.3e + 05 ± 7.0e + 04	65.0 ± 35.5	54.6
S22	4.2e + 06 ± 4.8e + 06	2.0 ± 2.3	114.1	3.0e + 06 ± 2.1e + 06	2.7 ± 1.9	69.1
S23	3.2e + 05 ± 3.7e + 05	26.2 ± 30.3	115.6	2.3e + 05 ± 1.6e + 05	36.1 ± 25.5	70.6
S24	8.9e + 05 ± 9.2e + 05	9.3 ± 9.6	103.2	6.5e + 05 ± 3.8e + 05	12.8 ± 7.5	58.2
S29	5.3e + 05 ± 5.7e + 05	15.8 ± 17.2	109.1	3.8e + 05 ± 2.4e + 05	21.7 ± 13.9	64.1
S31	7.6e + 05 ± 7.3e + 05	11.0 ± 10.6	96.4	5.5e + 05 ± 2.8e + 05	15.1 ± 7.8	51.4
S33	1.2e + 06 ± 1.3e + 06	6.7 ± 7.1	107.0	9.0e + 05 ± 5.6e + 05	9.2 ± 5.7	62.0
S38	7.6e + 04 ± 7.3e + 04	108.8 ± 103.7	95.4	5.5e + 04 ± 2.8e + 04	149.9 ± 75.5	50.4
S39	1.8e + 05 ± 1.7e + 05	47.0 ± 46.4	98.8	1.3e + 05 ± 6.9e + 04	64.7 ± 34.8	53.8
S42	2.3e + 06 ± 2.8e + 06	3.6 ± 4.4	122.7	1.7e + 06 ± 1.3e + 06	5.0 ± 3.9	77.7
S54	1.3e + 06 ± 2.5e + 06	6.3 ± 11.8	188.3	9.6e + 05 ± 1.4e + 06	8.7 ± 12.4	143.3
S55	6.8e + 04 ± 6.6e + 04	122.4 ± 119.4	97.6	4.9e + 04 ± 2.6e + 04	168.7 ± 88.7	52.6
S60	4.6e + 05 ± 4.5e + 05	17.9 ± 17.5	97.7	3.4e + 05 ± 1.8e + 05	24.6 ± 13.0	52.7
S66	6.0e + 06 ± 6.1e + 06	1.4 ± 1.4	101.4	4.4e + 06 ± 2.5e + 06	1.9 ± 1.1	56.4
S67	3.7e + 06 ± 3.7e + 06	2.2 ± 2.2	100.1	2.7e + 06 ± 1.5e + 06	3.1 ± 1.7	55.1
S71	9.2e + 05 ± 9.9e + 05	9.0 ± 9.7	107.3	6.7e + 05 ± 4.2e + 05	12.4 ± 7.7	62.3
S83	5.4e + 06 ± 6.0e + 06	1.5 ± 1.7	110.3	3.9e + 06 ± 2.6e + 06	2.1 ± 1.4	65.3
S85	1.6e + 07 ± 3.4e + 07	0.5 ± 1.1	211.0	1.2e + 07 ± 2.0e + 07	0.7 ± 1.2	166.0
S87	1.4e + 07 ± 1.5e + 07	0.6 ± 0.6	102.4	1.0e + 07 ± 6.0e + 06	0.8 ± 0.5	57.4
S89	2.5e + 06 ± 2.7e + 06	3.3 ± 3.6	107.8	1.8e + 06 ± 1.1e + 06	4.5 ± 2.9	62.8
S91	8.2e + 06 ± 8.3e + 06	1.0 ± 1.0	101.9	5.9e + 06 ± 3.4e + 06	1.4 ± 0.8	56.9
S96	5.9e + 06 ± 5.9e + 06	1.4 ± 1.4	100.0	4.3e + 06 ± 2.4e + 06	1.9 ± 1.1	55.0
S97	1.1e + 07 ± 1.3e + 07	0.8 ± 1.0	125.9	7.7e + 06 ± 6.2e + 06	1.1 ± 0.9	80.9
S145	3.1e + 06 ± 4.3e + 06	2.6 ± 3.6	136.7	2.3e + 06 ± 2.1e + 06	3.6 ± 3.3	91.7
S175	5.8e + 04 ± 6.4e + 04	143.4 ± 158.1	110.3	4.2e + 04 ± 2.7e + 04	197.6 ± 129.0	65.3
R34	5.4e + 06 ± 6.5e + 06	1.5 ± 1.8	120.5	3.9e + 06 ± 3.0e + 06	2.1 ± 1.6	75.5
R44	2.4e + 07 ± 3.7e + 07	0.4 ± 0.5	156.2	1.7e + 07 ± 1.9e + 07	0.5 ± 0.5	111.2

Conclusions

- Analysis of the stellar orbits around Sgr A* in the gravitational potentials with Yukawa-like corrections represents a powerful tool for studying a fifth force and constraining the graviton mass
- Fitting the simulated stellar orbits in Yukawa potential to the observed orbit of S-stars enabled us to estimate the range λ of a fifth force
- Assuming that λ corresponds to the Compton wavelength of graviton λ_g, we obtained new estimate for the upper bound for graviton mass that was consistent with the LIGO results, but obtained in an independent way
- Since 2019. our estimate is included in the *Gauge and Higgs Boson Particle Listings* published by PDG
- Extended matter may slightly increase these estimates for graviton mass, but within the expected interval
- We also studied modified and extended PPN formalisms and used them to further improve our constraints on strength and range of a fifth force, as well as on mass of graviton mass from the Schwarzschild precession in S2 star orbit detected by the GRAVITY Collaboration
- The obtained results showed that both massive graviton and a fifth force could play a fundamental role in modern physics, and thus it is of essential importance to further investigate these concepts

Instead of References: Impact of our results I

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REVIEW ARTICLE

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Experimental studies of black holes: status and future prospects

Reinhard Genzel[1,2,3] · **Frank Eisenhauer**[1,4] · **Stefan Gillessen**[1]

Further tests of General Relativity near a MBH GRAVITY Collaboration et al. (2019b) confirmed the Equivalence Principle in the orbit of S2 through a test of the linear positional invariance. In that paper the redshift data are split into spectroscopy of the hydrogen Brγ line and the HeI 2.1µm line, and the gravitational redshift term is computed for the two data sets. Einstein's Equivalence Principle stipulates that in free fall the motion should only depend on mass/energy, and not on composition. And indeed, GRAVITY Collaboration et al. (2019b) set an upper limit of a few 10^{-2} to the fractional difference of the gravitational redshift in hydrogen and helium. In another paper, Hees et al. (2017, 2020) used the Galactic Center data to set limits on a hypothetical fifth force, and variations in the fine structure constant. (Jovanović et al. 2024) analyzed the Schwarzschild precession of S2 in the framework of Yukawa gravity theory, and set an upper limit to the mass of the graviton, which is compatible with limits from aLIGO gravitational-wave data.

Instead of References: Impact of our results II

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Astronomy & Astrophysics

Improving constraints on the extended mass distribution in the Galactic center with stellar orbits

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Additionally, a deviation from general relativity, such as the one introduced by massive gravity theories or $f(R)$-gravity, could modify the gravitational potential through a Yukawa-like correction in the Newtonian limit, adding an additional precession of the stellar orbits to the prograde SP and the retrograde precession induced by an extended mass distribution (Hees et al. 2017; De Martino et al. 2021; Tan & Lu 2024; Jovanović et al. 2024a,b). For the specific case of massive gravity, the additional precession would be prograde and equal to $\delta\varphi_Y = \pi\sqrt{1-e^2}\frac{a^2}{\lambda^2}$ (Jovanović et al. 2024a), where $\lambda = \frac{\hbar}{m_g c}$ is the Compton wavelength of the massive graviton, m_g the mass of the graviton, and $\hbar$ the reduced Planck constant. From the observed precession of the S2 star, it is thus possible to derive a lower limit on λ and an upper limit on m_g, as is done in Hees et al. (2017); Jovanović et al. (2024a,b).

Instead of References: Impact of our results III

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Exploring the presence of a fifth force at the Galactic Center

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Instead of References: Impact of our results IV

- Part of the presented results is included in the White Paper of the COST action CA21136 *CosmoVerse*

Eleonora di Valentino,...,**Branko Dragovich, Duško Borka, Vesna Borka Jovanović, Goran S. Djordjević, Milan Milošević, Predrag Jovanović, Milan S. Dimitrijević, Milos Dordevic, Veljko Vujčić, Vladimir A. Srećković**, et al. arXiv:2504.01669 [astro-ph.CO] COST Action CA21136 - Addressing observational tensions in cosmology with systematics and fundamental physics (CosmoVerse)

arXiv:2504.01669v1 [astro-ph.CO]

The CosmoVerse White Paper: Addressing observational tensions in cosmology with systematics and fundamental physics

The CosmoVerse Network
(April 3, 2025)

4.3.1. Modified gravity in light of cosmic tensions

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