Avoiding singularities in Lorentzian-Euclidean black holes: The role of atemporality

Salvatore Capozziello

Nonlinearity, Nonlocality, and Ultrametricity
Belgrade, May 26-30, 2025

SSM
Scuola Superiore Meridionale

Based on

"Avoiding singularities in
Lorentzian-Euclidean black holes:
the role of atemporality",

S. Capozziello, S. De Bianchi, E. Battista,
ArXiv: 2404.17267, published in
Physical Review D 109 (2024) 10, 104060

SUMMARY

We investigate a Schwarzschild metric exhibiting a signature change across the event horizon, which gives rise to what we term a Lorentzian-Euclidean black hole. The resulting geometry is regularized employing the Hadamard *partie finie* technique, which allows us to prove that the metric represents a solution of vacuum Einstein equations. In this framework, we introduce the concept of atemporality as the dynamical mechanism responsible for the transition from a regime with a real-valued time variable to a new one featuring an imaginary time. We show that this mechanism prevents the occurrence of the singularity and discuss that, thanks to the regularized Kretschmann invariant, the atemporality can be considered as a characteristic feature of this black hole.

OUTLINE

1. *SIGNATURE-CHANGING METRICS*

2. *JUNCTION CONDITIONS AND THIN SHELLS*

3. *THE LORENTZIAN-EUCLIDEAN BLACK HOLE METRIC*

4. *THE REGULARIZATION PROCESS*

5. *AVOIDING THE SINGULARITY*

6. *CONCLUSIONS*

SIGNATURE-CHANGING METRICS

- **Metrics whose signature changes from Lorentzian to Euclidean one and vice versa:**

-Studied in classical and quantum General Relativity (GR)

- **Quantum GR:**

- Quantum cosmology →
- **Hartle-Hawking no-boundary conditions**
- **Linde proposal**
- **Vilenkin proposal (tunneling from nothing)**

-Loop quantum cosmology

-Supergravity and String theory

SIGNATURE-CHANGING METRICS

- **<u>Classical GR:</u>**

-Not forbidden by Einstein field equations

-Homogeneous and isotropic
Friedmann-Lemaître-Robertson-Walker geometries

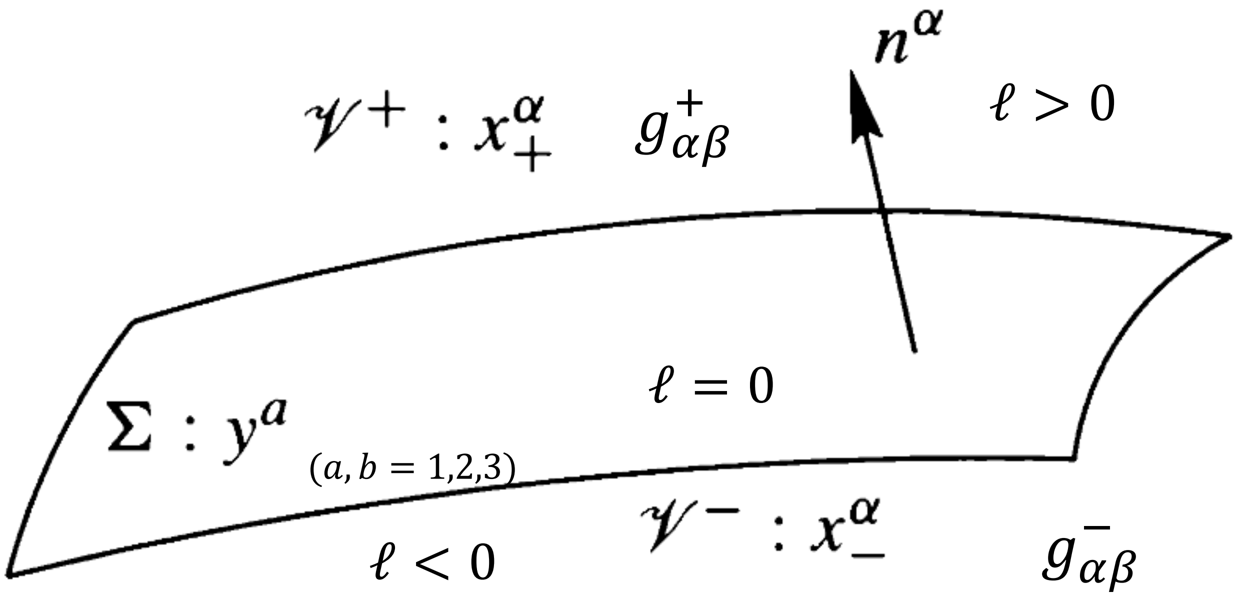
i. **Similar properties with quantum scenarios satisfying the Hartle-Hawking no-boundary conditions**

ii. **Related to the tunneling solutions of the Wheeler-DeWitt equation in** ***Q*****uantum** ***C*****osmology**

JUNCTION CONDITIONS AND THIN SHELLS

- **Joining two metrics at a common boundary, which divides the spacetime into two distinct regions**

Israel-Barrabes formalism (metrics with unchanging signature)



$n_\mu = \alpha \partial_\mu \ell$

Σ is timelike ($\alpha = 1$) or spacelike ($\alpha = -1$)

The same coordinates y^a settled on both sides of Σ

$$g_{\mu\nu} = \Theta(\ell) g_{\mu\nu}^+ + \Theta(-\ell) g_{\mu\nu}^-$$

metric in coordinates x^μ

JUNCTION CONDITIONS AND THIN SHELLS

What conditions have to be imposed on the metric so that $g_{\alpha\beta}$ gives a distribution-valued solution of Einstein field equations?

Junction conditions that involve three-tensors on Σ

$$[F] := F|_+ - F|_-$$

Jump discontinuity of any tensorial quantity F across Σ

$$[F] = 0$$

F is continuous at Σ

$$[F] \neq 0$$

F is discontinuous across Σ;
$[F]$ is the jump discontinuity of F across Σ

In our hypotheses $[n^\alpha] = [x^\alpha] = [y^a] = 0$

$$g_{\mu\nu,\gamma} = \Theta(\ell) g^+_{\mu\nu,\gamma} + \Theta(-\ell) g^-_{\mu\nu,\gamma} + \alpha\delta(\ell)[g_{\mu\nu}] n_\gamma$$

JUNCTION CONDITIONS AND THIN SHELLS

- **First junction condition: the metric is continuous across Σ**

$$[g_{\mu\nu}] = 0$$

In the coordinate system x^{α}

$$[h_{ab}] = 0$$

Induced metric (coordinate y^{a})

coordinate-invariant statement

Metric tangential derivatives are also continuous, but the normal derivatives are not:

$$[g_{\alpha\beta,\gamma}] = \kappa_{\alpha\beta} n_{\gamma}$$

JUNCTION CONDITIONS AND THIN SHELLS

- δ**-function part of the Riemann tensor**

$$A^{\alpha}_{\beta\gamma\delta} = \frac{\alpha}{2}\left(\kappa^{\alpha}_{\delta} n_{\beta} n_{\gamma} - \kappa^{\alpha}_{\gamma} n_{\beta} n_{\delta} - \kappa_{\beta\delta} n^{\alpha} n_{\gamma} + \kappa_{\beta\gamma} n^{\alpha} n_{\delta}\right)$$

- δ**-function part of the Ricci tensor**

$$A_{\alpha\beta} \equiv A^{\mu}_{\alpha\mu\beta} = \frac{\alpha}{2}\left(\kappa_{\mu\alpha} n^{\mu} n_{\beta} + \kappa_{\mu\beta} n^{\mu} n_{\alpha} - \kappa^{\mu}_{\mu} n_{\alpha} n_{\beta} - \alpha \kappa_{\alpha\beta}\right)$$

- δ**-function part of the Ricci scalar**

$$A \equiv A^{\alpha}_{\alpha} = \alpha\left(\kappa_{\mu\nu} n^{\mu} n^{\nu} - \alpha \kappa^{\mu}_{\mu}\right)$$

JUNCTION CONDITIONS AND THIN SHELLS

Einstein field equations give the following expression for the stress-energy tensor:

$$T_{\alpha\beta} = \theta(\ell)T^{+}_{\alpha\beta} + \theta(-\ell)T^{-}_{\alpha\beta} + \delta(\ell)S_{\alpha\beta}$$

with $8\pi S_{\alpha\beta} = A_{\alpha\beta} - \frac{1}{2}Ag_{\alpha\beta}$

The δ-function term of $T_{\alpha\beta}$ is associated with the presence of a thin distribution of matter, which is referred to as surface layer or thin shell

The stress-energy tensor of the thin shell is $S_{\alpha\beta}$

JUNCTION CONDITIONS AND THIN SHELLS

Explicitly, the thin shell stress-energy tensor depends on the jump discontinuity of the extrinsic curvature tensor K_{ab} of Σ:

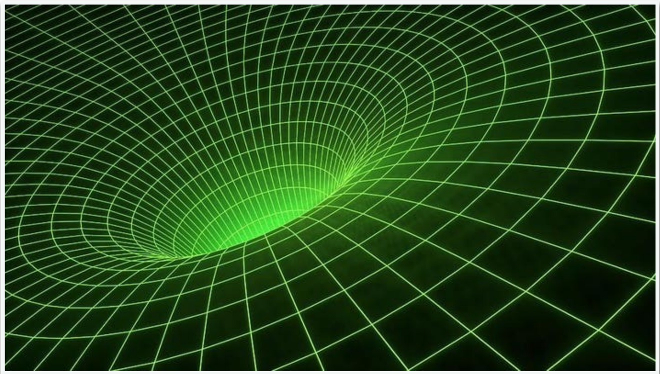
$$S_{ab} = -\frac{\alpha}{8\pi}\left([K_{ab}] - [K]h_{ab}\right)$$

- **Second junction condition: $[K_{ab}] = 0$, which implies $A^{\alpha}_{\beta\gamma\delta} = 0$**

When junction conditions are satisfied, then the two metrics $g^{\pm}_{\mu\nu}$ can be joined smoothly through Σ

JUNCTION CONDITIONS AND THIN SHELLS

- **When Σ is either spacelike or timelike, then only the Ricci part of the Riemann tensor can show a distributional singularity**



- **When Σ is null, then both the Ricci and Weyl part of the Riemann tensor can present Dirac-delta singularities**

Thin shell

Impulsive gravitational wave

LORENTZIAN-EUCLIDEAN BLACK HOLE

Lorentzian-Euclidean Schwarzschild metric in standard coordinates $\{t, r, \theta, \phi\}$

$$ds^2 = -\varepsilon\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r^2 d\Omega^2,$$

where

$$\varepsilon = \text{sign}\left(1 - \frac{2M}{r}\right) = 2H\left(1 - \frac{2M}{r}\right) - 1,$$

Sign function

Step function

H(0)=1/2

LORENTZIAN-EUCLIDEAN BLACK HOLE

Therefore, the spacetime manifold is divided as $V = V_+ \cup V_-$ and

- $\varepsilon = 1$ **if $r > 2M$: Lorentzian signature** $(-+++)$
- $\varepsilon = 0$ **if $r = 2M$: metric is degenerate** $\det g_{\mu\nu} = 0$
- $\varepsilon = -1$ **if $r < 2M$: metric has an Euclidean structure and attains ultrahyperbolic signature** $(--++)$
- $\Sigma : r = 2M$ **change surface (null hypersurface)**
- **Metric and its derivatives are discontinuous across the change surface**

$$[g_{\alpha\beta}] \neq 0 \qquad [g_{\alpha\beta,\mu}] \neq 0$$

LORENTZIAN-EUCLIDEAN BLACK HOLE

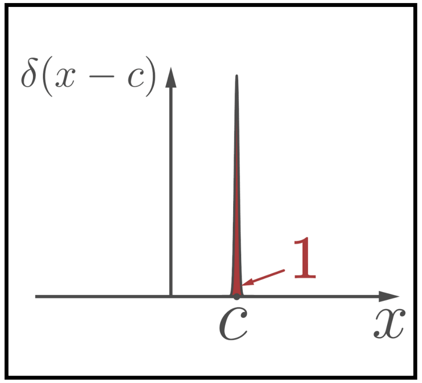
Metric in Gullstrand-Painlevé coordinates $(\mathcal{T}, r, \theta, \phi)$

$$ds^2 = -\varepsilon d\mathscr{T}^2 + \left(dr + \sqrt{\varepsilon}\sqrt{\frac{2M}{r}}d\mathscr{T}\right)^2 + r^2 d\Omega^2.$$

The only pathology is related to the fact that the metric becomes degenerate on the change surface Σ**, i.e., when** $r = 2M$ **and** $\varepsilon = 0$

THE REGULARIZATION PROCESS

Recall that $[g_{\alpha\beta}] \neq 0$ and $[g_{\alpha\beta,\mu}] \neq 0$ → first junction condition cannot be satisfied



- **Dirac-delta-like contributions arising in the Riemann tensor**
- **Terms proportional to ε', $(\varepsilon')^2$, ε'' $\Rightarrow$ Linear and quadratic terms in the Dirac-delta function $\delta(r-2M)$ in the Riemann tensor**

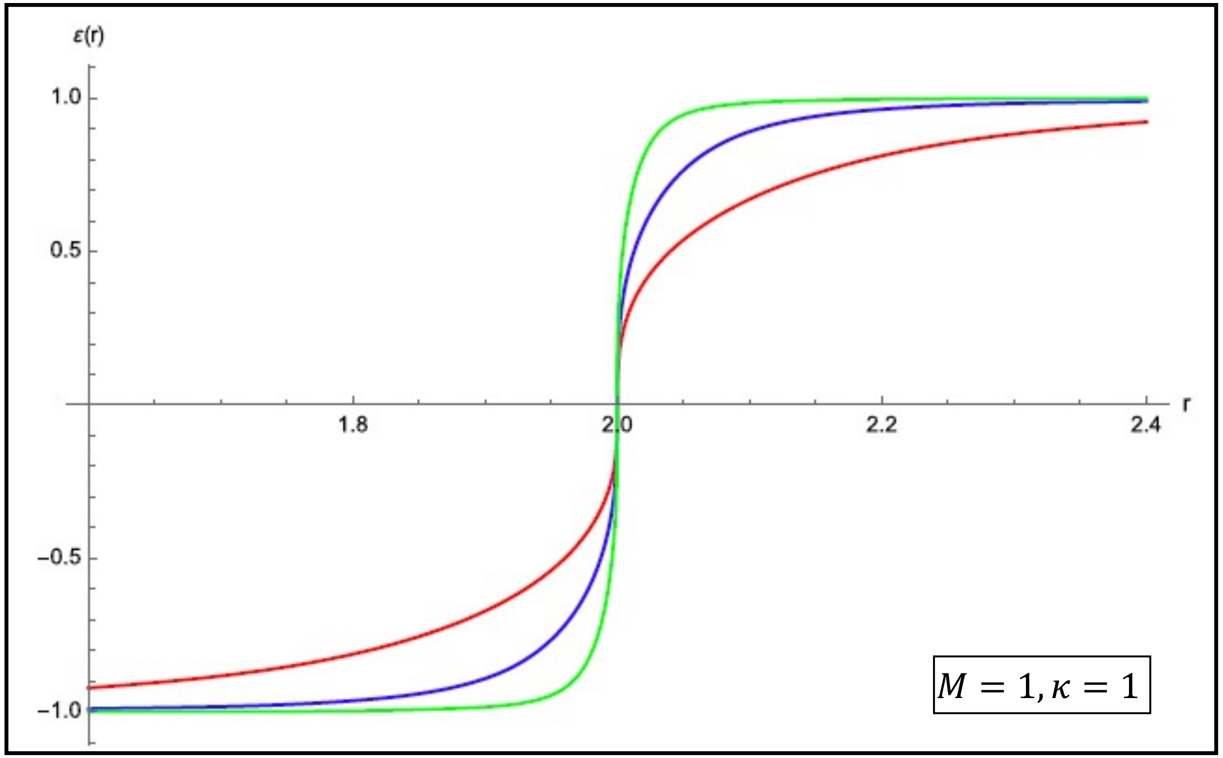
Proper regularization scheme

THE REGULARIZATION PROCESS

- **Smooth approximation of** $\varepsilon(r) = 2H(1 - 2M/r) - 1$**:**

$$\varepsilon(r) = \frac{(r-2M)^{1/(2\kappa+1)}}{[(r-2M)^2+\rho]^{1/2(2\kappa+1)}},$$

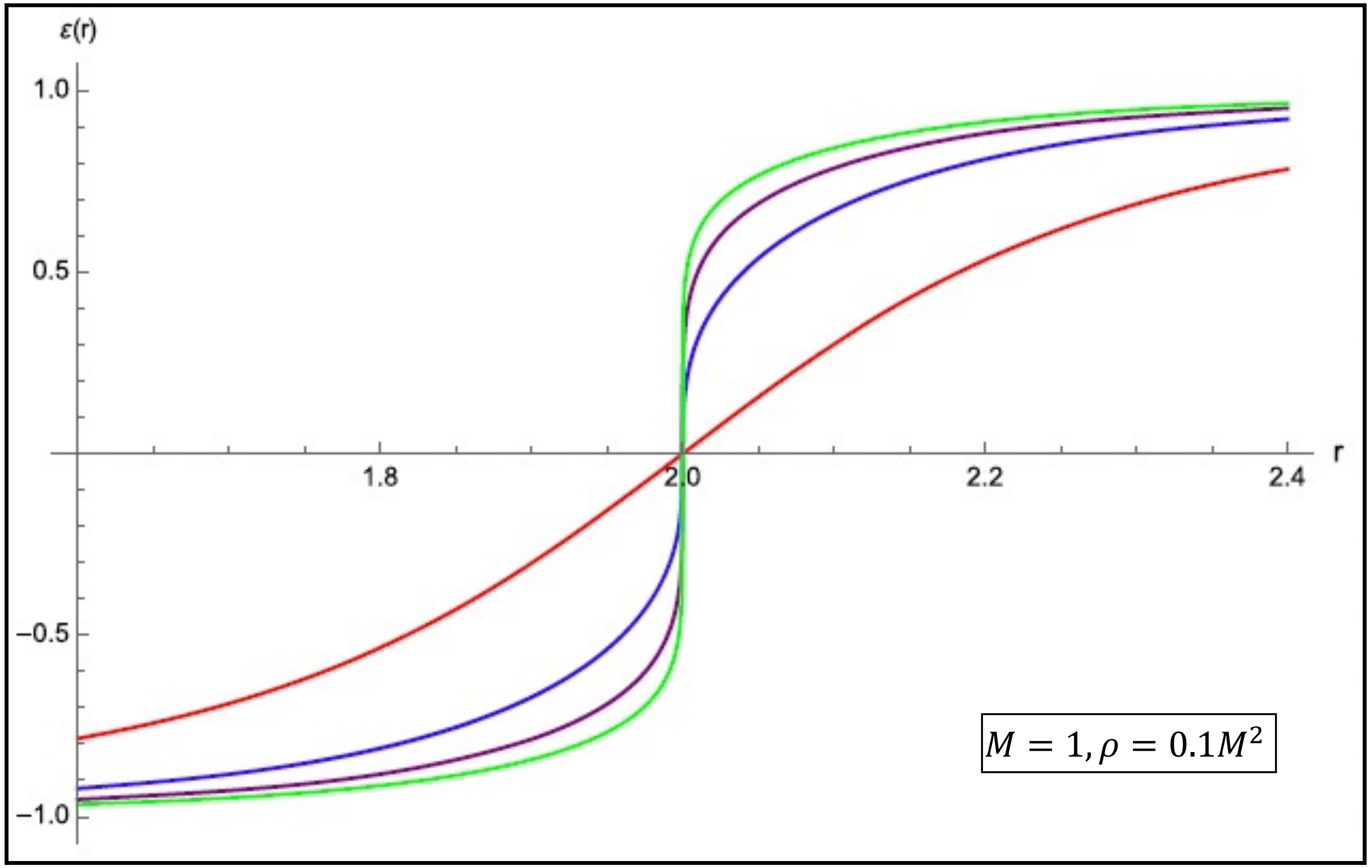
ρ/M^2: **small positive quantity**
κ: **positive integer**



The smaller ρ/M^2**, the sharper** $\varepsilon(r)$

$\rho = 0.1M^2$: **red curve**
$\rho = 0.01M^2$: **blue curve**
$\rho = 0.001M^2$: **green curve**

THE REGULARIZATION PROCESS



The larger κ, the steeper $\varepsilon(r)$

$\kappa = 0$: **red curve**
$\kappa = 1$: **blue curve**
$\kappa = 2$: **purple curve**
$\kappa = 3$: **green curve**

We will see that our regularization scheme requires $\kappa \geq 1$

THE REGULARIZATION PROCESS

- **The Riemann tensor contains linear-in-delta ill-defined terms of the type**

$$\int \mathrm{d}r \frac{\delta(r-2M)}{\varepsilon(r)},$$

Hadamard *partie finie* regularization method & approximation of $\varepsilon(r)$

$$\frac{\delta(x)}{|x|^n} \equiv 0,$$

n**: positive integer**

$x := r - 2M$

THE REGULARIZATION PROCESS

- **Let $F(\xi; a)$ be a function of ξ which diverges as ξ approaches a. We assume that near $\xi = a$**

$$F(\xi; a) = \sum_{n=0}^{n_{\max}} s^{-n} f_n(s; a) + O(s), \qquad s = |\xi - a|$$

- **The function diverges as $s^{-n_{\max}}$ when $\xi \to a$ and has no well-defined value at $\xi = a$**

- **We can regularize it by extracting its *partie finie* at the singular point $\xi = a$, which is defined by**

$$\langle F \rangle(a) := \frac{1}{2\pi} \int_0^{2\pi} f_0(s; a) \mathrm{d}\theta$$

Angular average of the zeroth term $f_0(s; a)$ of the Laurent series

THE REGULARIZATION PROCESS

- **The *partie finie* can be used to make sense of the product of F with the delta function $\delta(\xi - a)$, since we declare that**

$$F(\xi; a)\delta(\xi - a) \equiv \langle F \rangle(a)\delta(\xi - a)$$

$$\int F(\xi; a)\delta(\xi - a)\mathrm{d}\xi = \langle F \rangle(a)$$

- **In our case** $\dfrac{\delta(x)}{|x|^n} \equiv 0,$

$$F = |x|^{-n} := |r - 2M|^{-n}$$

$$\langle F \rangle = 0$$

THE REGULARIZATION PROCESS

- **Quadratic-in-delta ill-defined terms occurring in the Riemann tensor**

Regularized within our model since their coefficients vanish when $r = 2M$

Sad Riemann

Terms as $\delta^2(r - 2M)$ give vanishing contribution in the distributional sense to the Riemann tensor

Happy Riemann

THE REGULARIZATION PROCESS

- **An example: regularization of $R^{r}_{r\mathcal{T}r}$**

$$R^{r}_{r\mathcal{T}r} = \sqrt{\frac{M}{r}}\,\frac{r^2(2M-r)\varepsilon'^2 + 2r\varepsilon[r(r-2M)\varepsilon'' + 3M\varepsilon'] - 8M\varepsilon^2}{2\sqrt{2}r^3\varepsilon^{3/2}}$$

-Terms linear in $\varepsilon'(x)$ yield an integral proportional to ($x := r - 2M$)

$$\int \mathrm{d}x \frac{\delta(x)}{\varepsilon^{1/2}} = \int \mathrm{d}x \delta(x) \frac{(x^2+\rho)^{1/4(2\kappa+1)}}{x^{1/2(2\kappa+1)}}$$

$$= \int \mathrm{d}x \left(\frac{\delta(x)}{x^p x^{1/2(2\kappa+1)}}\right)\left[x^p (x^2+\rho)^{1/4(2\kappa+1)}\right]$$

Approximation for $\varepsilon(r)$

$\delta(x)/|x|^n \equiv 0$
(Hadamard prescription)

vanishing in $x = 0$

THE REGULARIZATION PROCESS

-Terms depending on $(\varepsilon')^2$ lead to an integral proportional to

$$\int \mathrm{d}x \frac{x\delta^2(x)}{\varepsilon^{3/2}} = \int \mathrm{d}x\delta^2(x)(x^2+\rho)^{3/4(2\kappa+1)}x^{(4\kappa-1)/2(2\kappa+1)},$$

Vanishing contribution in the distributional sense as the coefficient of $\delta^2(x)$ is zero in $x=0$ if we suppose $\kappa \geq 1$

-Terms depending on ε'' give an integral proportional to

$$\int \mathrm{d}x \frac{x\varepsilon''(x)}{\varepsilon^{1/2}} = 2\int \mathrm{d}x \frac{x\delta'(x)}{\varepsilon^{1/2}} == -2\int \mathrm{d}x\delta(x)\frac{(x^2+\rho)^{1/4(2\kappa+1)}}{x^{1/2(2\kappa+1)}}$$

$$+2\int \mathrm{d}x\delta^2(x)x\frac{(x^2+\rho)^{3/4(2\kappa+1)}}{x^{3/2(2\kappa+1)}},$$

Vanishing contribution in the distributional sense

$\delta(x)/|x|^n \equiv 0$

(Hadamard prescription)

THE REGULARIZATION PROCESS

The regularized $R^{r}_{r\mathcal{T}r}$ assumes this form

$$R^{r}_{r\mathcal{T}r} = -2\sqrt{2}\left(\frac{M}{r}\right)^{3/2}\frac{\sqrt{\varepsilon}}{r^2}$$

Remaining regularized Riemann tensor components read as

$$\begin{aligned}
R^{r}{}_{\theta\theta r} &= \frac{M}{r}, \\
R^{r}{}_{\phi\phi r} &= \sin^2\theta\, R^{r}{}_{\theta\theta r}, \\
R^{r}{}_{\mathcal{T}\mathcal{T}r} &= \frac{2M\varepsilon(r-2M)}{r^4}, \\
R^{\theta}{}_{r\theta r} &= -\frac{1}{r^2}R^{r}{}_{\theta\theta r}, \\
R^{\theta}{}_{r\mathcal{T}\theta} &= -\frac{1}{2}R^{r}{}_{r\mathcal{T}r}, \\
R^{\theta}{}_{\phi\phi\theta} &= -2\sin^2\theta\, R^{r}{}_{\theta\theta r},
\end{aligned}$$

$$\begin{aligned}
R^{\theta}{}_{\mathcal{T}\theta r} &= \frac{1}{2}R^{r}{}_{r\mathcal{T}r}, \\
R^{\theta}{}_{\mathcal{T}\mathcal{T}\theta} &= -\frac{1}{2}R^{r}{}_{\mathcal{T}\mathcal{T}r} \\
R^{\phi}{}_{r\phi r} &= -\frac{1}{r^2}R^{r}{}_{\theta\theta r}, \\
R^{\phi}{}_{r\mathcal{T}\phi} &= -\frac{1}{2}R^{r}{}_{r\mathcal{T}r}, \\
R^{\phi}{}_{\theta\phi\theta} &= 2R^{r}{}_{\theta\theta r}, \\
R^{\phi}{}_{\mathcal{T}\phi r} &= \frac{1}{2}R^{r}{}_{r\mathcal{T}r},
\end{aligned}$$

$$\begin{aligned}
R^{\phi}{}_{\mathcal{T}\mathcal{T}\phi} &= -\frac{1}{2}R^{r}{}_{\mathcal{T}\mathcal{T}r}, \\
R^{\mathcal{T}}{}_{r\mathcal{T}r} &= \frac{2}{r^2}R^{r}{}_{\theta\theta r}, \\
R^{\mathcal{T}}{}_{\theta\mathcal{T}\theta} &= -R^{r}{}_{\theta\theta r}, \\
R^{\mathcal{T}}{}_{\phi\mathcal{T}\phi} &= -\sin^2\theta R^{r}{}_{\theta\theta r}, \\
R^{\mathcal{T}}{}_{\mathcal{T}\mathcal{T}r} &= -R^{r}{}_{r\mathcal{T}r}.
\end{aligned}$$

THE REGULARIZATION PROCESS

-The regularized Riemann tensor does not depend on the Dirac-delta function and it is discontinuous across Σ, as $[R^{\alpha}_{\beta\gamma\delta}] \neq 0$

-The ensuing Ricci tensor, Ricci scalar, and consequently Einstein tensor vanish

↓

Σ does not represent a thin shell

- The regularized Kretschmann invariant is

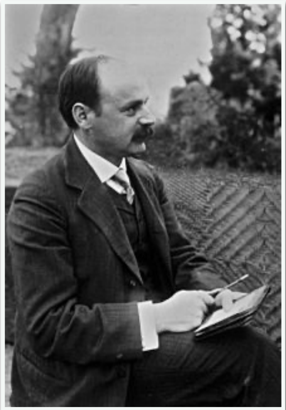
$$R_{\alpha\beta\gamma\mu}R^{\alpha\beta\gamma\mu} = \frac{48M^2}{r^6}$$

↓

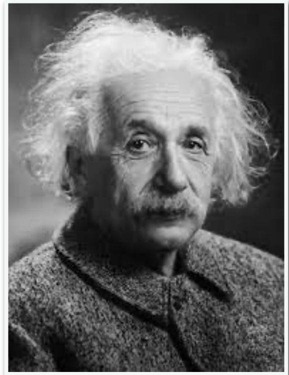
Σ does not give rise to a new curvature singularity

THE REGULARIZATION PROCESS

-The Weyl tensor stemming from the regularized Riemann tensor is discontinuous across Σ, but it does not depend on Dirac-delta function



No impulsive gravitational wave on Σ



The Lorentzian-Euclidean Schwarzschild metric is a valid signature-changing solution of vacuum Einstein field equations

AVOIDING THE SINGULARITY

Henceforth, we will use the Schwarzschild coordinates $\{t, r, \theta, \phi\}$

$$\mathrm{d}s^2 = -\varepsilon\left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\left(1 - \frac{2M}{r}\right)} + r^2\mathrm{d}\Omega^2,$$

with

$\varepsilon = 1$ **if** $r > 2M$**,** $\varepsilon = 0$ **if** $r = 2M$**, and** $\varepsilon = -1$ **if** $r < 2M$**.**

Let us study the motion of bodies radially approaching the Lorentzian-Euclidean black hole

AVOIDING THE SINGULARITY

- **Geodesic motion**

-Observer starting at rest at some finite distance $r_i > 2M$

-Describe the radial variable via the relation

$$r(\eta) = r_i \cos^2(\eta/2), \eta \in [0, \eta_H]$$

-Equations governing infalling radial geodesics are

$$\dot{r} = -\sqrt{\frac{\varepsilon^4 \sin^2(\eta/2) + E^2[\cos^2(\eta/2) - \varepsilon^4]}{\varepsilon^3 \cos^2(\eta/2)}}$$

$$\dot{t} = \frac{E}{\varepsilon^2} \frac{\cos^2(\eta/2)}{\cos^2(\eta/2) - (1 - E^2)}$$

AVOIDING THE SINGULARITY

along with

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\eta} = \left(\dot{r}\right)^{-1}\frac{\mathrm{d}r}{\mathrm{d}\eta} = r_i \sin(\eta/2)\cos^2(\eta/2)\sqrt{\frac{\varepsilon^3}{\varepsilon^4\sin^2(\eta/2) + E^2[\cos^2(\eta/2) - \varepsilon^4]}}$$

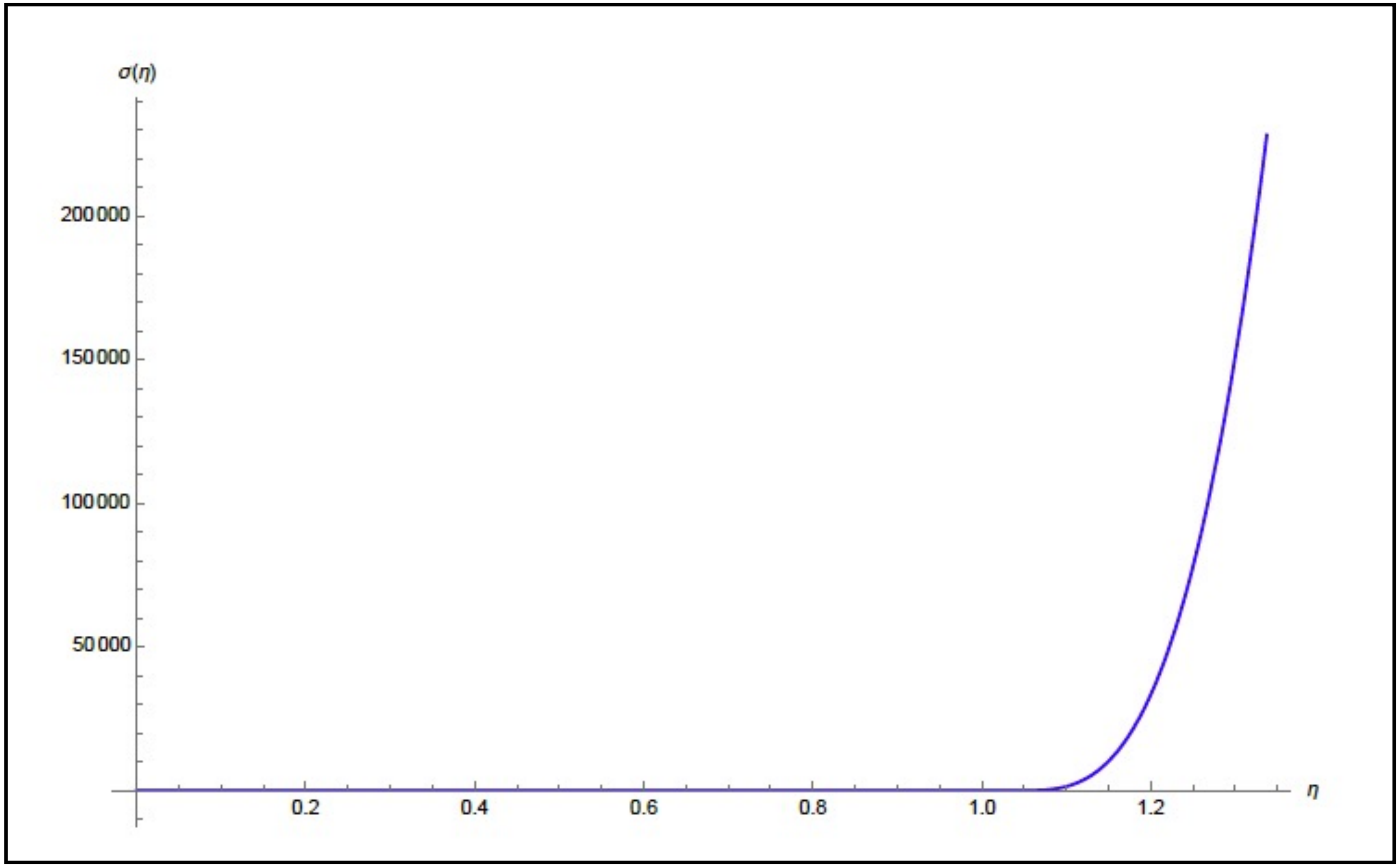
$$\frac{\mathrm{d}t}{\mathrm{d}\eta} = \dot{t}\,\frac{\mathrm{d}\sigma}{\mathrm{d}\eta} = \frac{E}{\varepsilon^2}\,\frac{r_i\cos^4(\eta/2)\sin(\eta/2)}{\cos^2(\eta/2) - (1 - E^2)}\sqrt{\frac{\varepsilon^3}{\varepsilon^4\sin^2(\eta/2) + E^2[\cos^2(\eta/2) - \varepsilon^4]}}$$

-The radial velocity $\dot{r}$, and the derivatives $\mathrm{d}\sigma/\mathrm{d}\eta, \mathrm{d}t/\mathrm{d}\eta$ **assume imaginary values as** $r < 2M$

- The radial velocity $\dot{r}$ vanishes at $r = 2M$

AVOIDING THE SINGULARITY

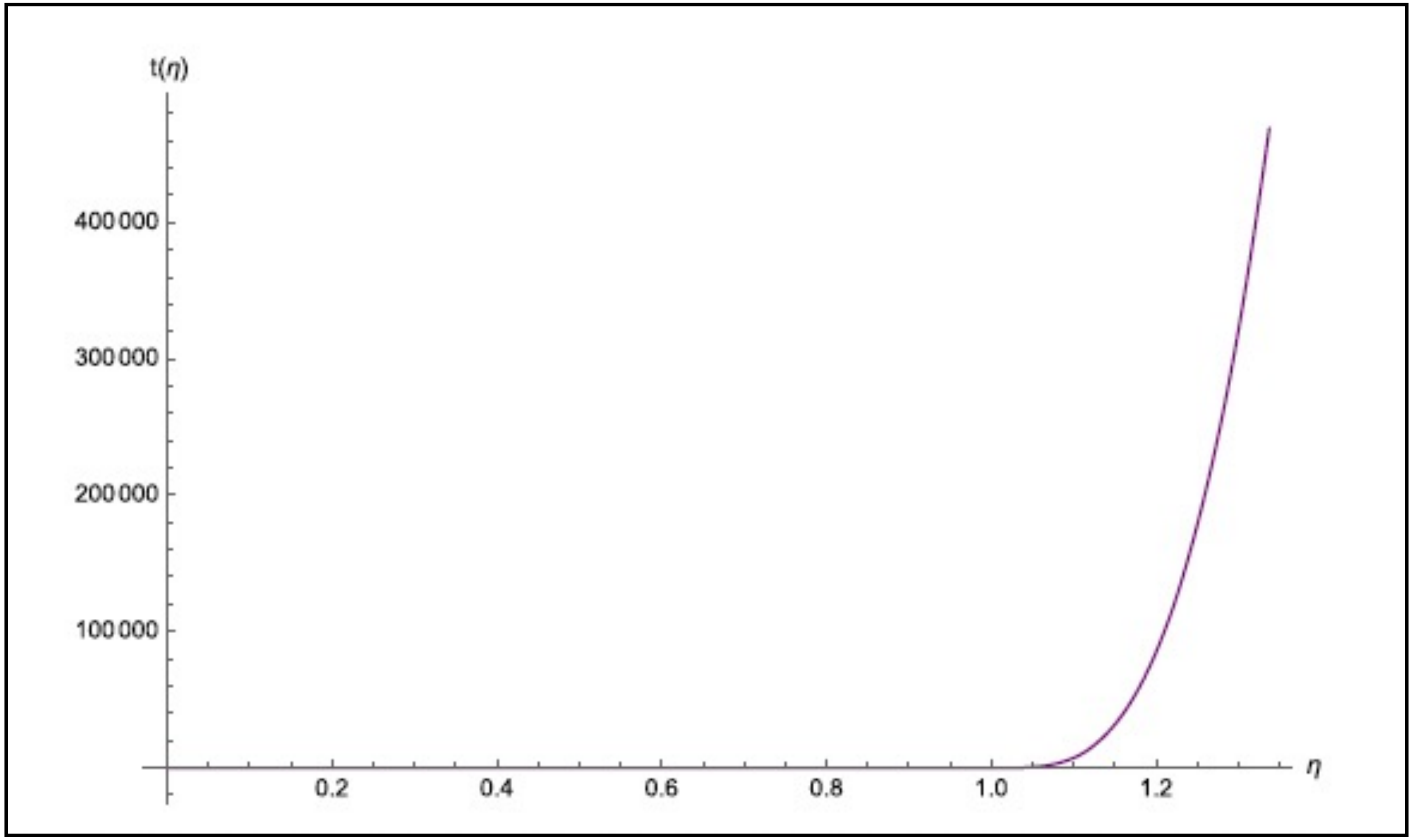
-The observer in radial free fall takes an infinite amount of proper time σ to stop at the event horizon



event horizon:
$\eta \approx 1.3$

AVOIDING THE SINGULARITY

-The observer in radial free fall takes an infinite amount of time to stop at the event horizon also from the point of view of an observer stationed at infinity



event horizon:
$\eta \approx 1.3$

AVOIDING THE SINGULARITY

- **Accelerated motion**

-Radially accelerated observer whose trajectory begins at rest from a large distance from the black hole

$$a^{\lambda} = \frac{\mathrm{d}U^{\lambda}}{\mathrm{d}\sigma} + \Gamma^{\lambda}_{\mu\nu} U^{\mu} U^{\nu}$$

$$U^{\mu} := \frac{\mathrm{d}x^{\mu}}{\mathrm{d}\sigma}$$

-Radial-directed orbit (θ, ϕ constant)

$$a^{t} = \frac{\mathrm{d}U^{t}}{\mathrm{d}\sigma} + 2\Gamma^{t}_{tr} U^{t} U^{r}$$

$$a^{r} = \frac{\mathrm{d}U^{r}}{\mathrm{d}\sigma} + \Gamma^{r}_{tt} U^{t} U^{t} + \Gamma^{r}_{rr} U^{r} U^{r}$$

Christoffel symbols result regularized via our technique

AVOIDING THE SINGULARITY

- **Radial velocity**

$$U^r = -\sqrt{\varepsilon}\sqrt{\mathcal{F}^2 - (1 - 2M/r)}$$

$$\mathcal{F} = f(\sigma)\sqrt{1 - 2M/r},$$

$$f(\sigma) > 1$$

↓

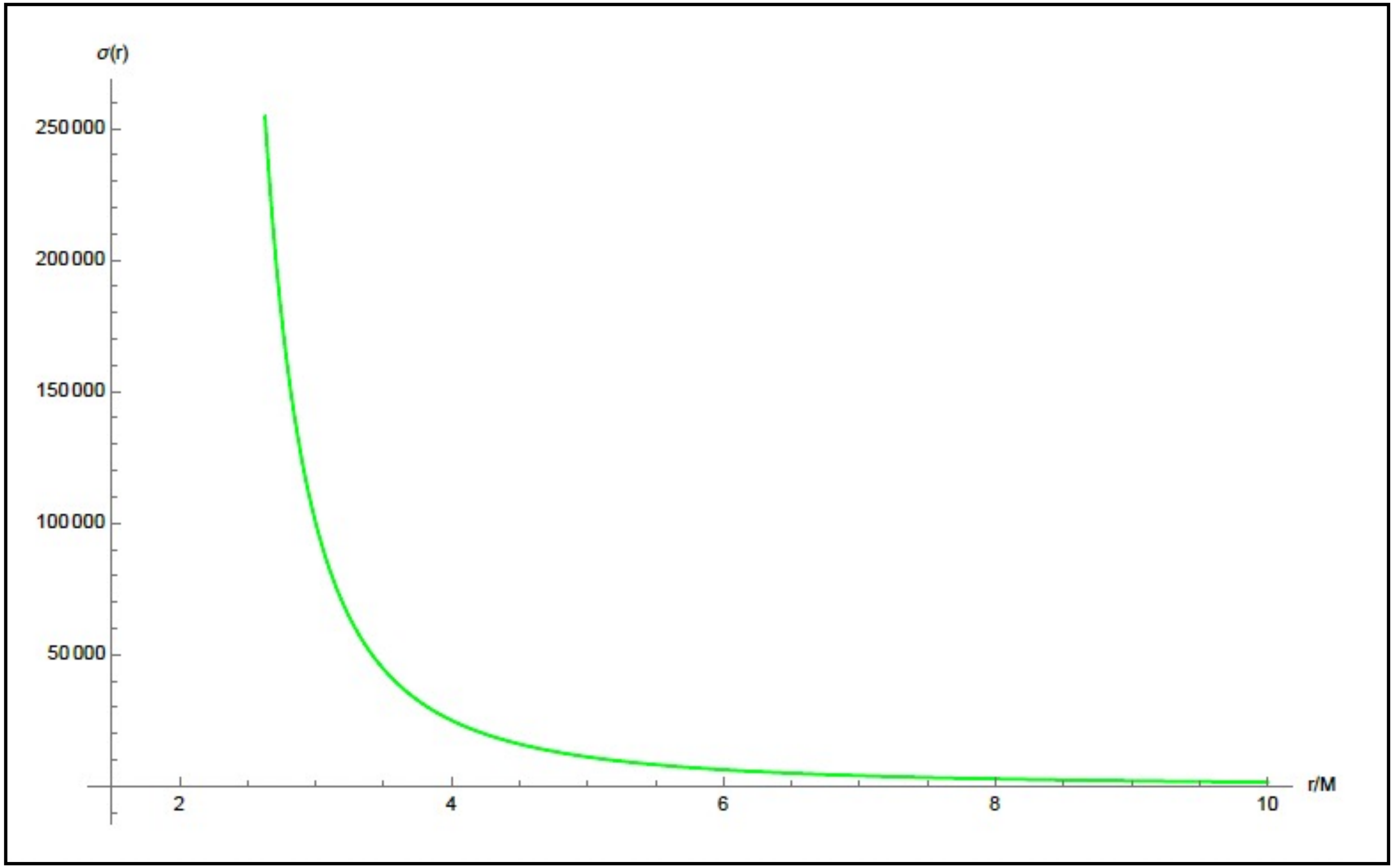
U^r **vanishes on the event horizon and becomes imaginary inside it**

-Differential equation for the proper time σ

$$\frac{\mathrm{d}\sigma}{\mathrm{d}r} = -\frac{1}{\sqrt{\varepsilon[\mathcal{F}^2 - (1-2M/r)]}}$$

AVOIDING THE SINGULARITY

The accelerated observer takes an infinite amount of proper time σ to stop at the event horizon



Discussion & Conclusions

- **The signature change of the Lorentzian-Euclidean metric can be ascribed to the emergence of an imaginary time variable t when $r < 2M$. We propose to relate this feature to the concept of "*ATEMPORALITY*"**

↓

Atemporality configures* as the dynamical *mechanism by which an observer pointing towards the event horizon cannot reach the singularity in r = 0, because real-valued geodesics and accelerated orbits cannot be prolonged up to there.

As a consequence, both time variable and radial velocity become imaginary inside the black hole. The parameter "measuring" the degree of atemporality is the Kretschmann scalar

$$K(r = 2M) = \frac{3}{4M^4},$$

which is related to the mass *M* of the black hole.

Discussion & Conclusions

- **There exists an analogy between atemporality and the tunnelling effect in Quantum Mechanics.**

Quantum Mechanics: the nature of the quantum wave function changes inside and outside the potential barrier.

Atemporality: the nature of time, as well as that of geodesics and accelerated paths, changes in passing through the event horizon.

↓

***Atemporality* consists in the change of dynamical behavior.**

Discussion & Conclusions

- **There is no preference between a real-valued and imaginary time variable**

Hawking himself has stated it in his popular science book “*The Universe in a Nutshell*”:

“One might think this means that imaginary numbers are just a mathematical game having nothing to do with the real world. From the viewpoint of positivist philosophy, however, one cannot determine what is real. All one can do is to find which mathematical models describe the universe we live in. It turns out that a mathematical model involving imaginary time predicts not only effects we have already observed but also effects we have not been able to measure yet nevertheless believe in for other reasons.”

Discussion & Conclusions

• Atemporality represents a limit for *measurements* and prevents the loss of *causality*:

- Causality is lost when time becomes imaginary
- Our system is geodesically complete
- Measurement cannot be performed *inside* a black hole

↓

Similarities with Uncertainty Principle

Discussion & Conclusions

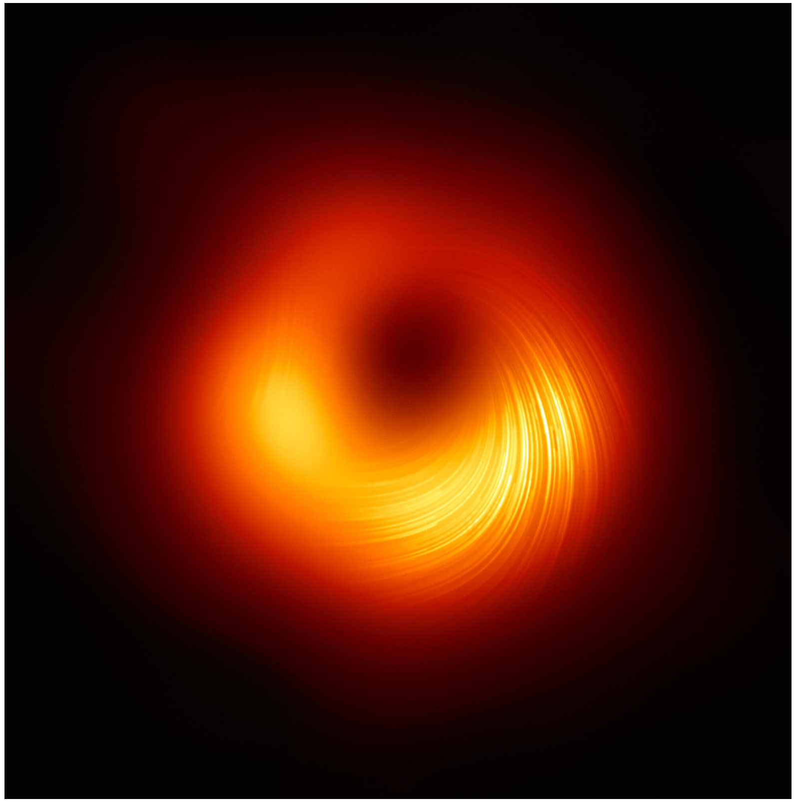
- **Atemporality ensures that conservation laws are not violated in the Lorentzian region and at the event horizon: the velocity of the infalling particle vanishes at the event horizon and becomes imaginary after having crossed it.**

The time translation symmetry and the related conservation of energy can hold if and only if the singularity at *r=0* can be evaded.

- **Bunch of particles (massive and massless) accumulate on the event horizon:**

  Can this fact be the observational feature of the model?

Discussion & Conclusions



In our approach, the bunch of particles accumulating around the event horizon, could shape the luminous silouhette around the black hole. Forthcoming observational campaigns could probe this statement.

Work in progress!

Happy Birthday,
Dear Branko!