Self-accelerating Scorer beams

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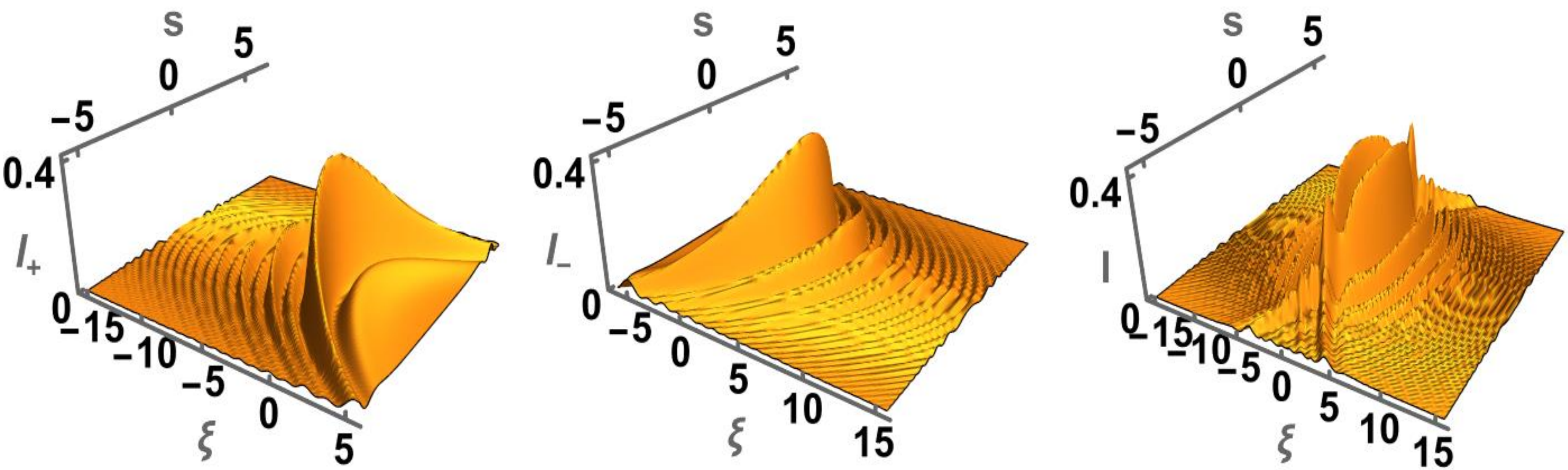


NONLINEARITY, NONLOCALITY AND ULTRAMETRICITY
BELGRADE, MAY 26-30 2025, SERBIA

Celebrating 80 years of Branko Dragovich

Summary: *This talk is dedicated to honoring Branko Dragovich on his 80th birthday. I met Branko in 1981, when I joined the Institute of Physics in Belgrade and he was a young dynamic director of the Institute of Theoretical Physics there. Back then, the Institute of Physics was scattered across five different locations in the city, but once it was united under one roof in 1983, it was a smooth successful sailing from there!*

Dear Branko, on this special occasion I wish you a very Happy Birthday and **многая лета***!*

The talk iself explores the fascinating world of accelerating beams, with a focus on Airy and Scorer beams – a recent discovery by WP Zhong and M. Belic.

Outline

- Accelerating light beams?
- Quantum-mechanical Airy wave-packets
- Nonlinear Airy beams in optics
- Self-accelerating Scorer beams
- Summary

Light travels in straight lines

Courtesy of D. Christodoulides

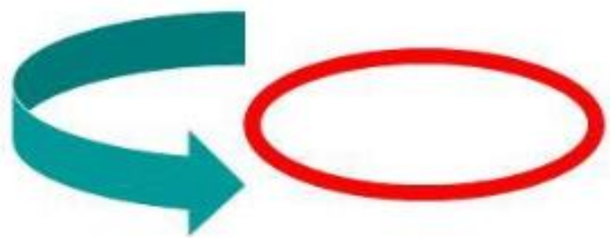
Euclid of Alexandria
325-265 BC

Euclid's Optics
~ 300 BC

Euclid's Geometry:
The Elements

Yes, but…

Yet, can light bend ??

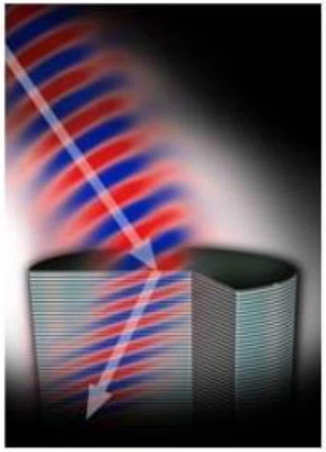


In arts – yes!
In physics?

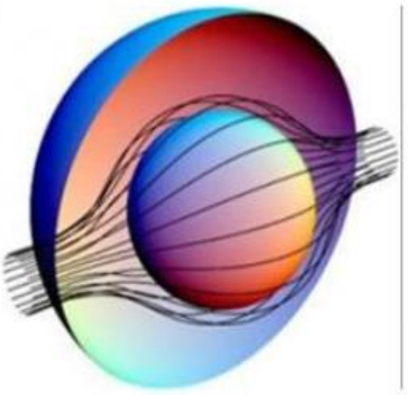
Can light bend ?? Sure!

Refraction

Negative refraction



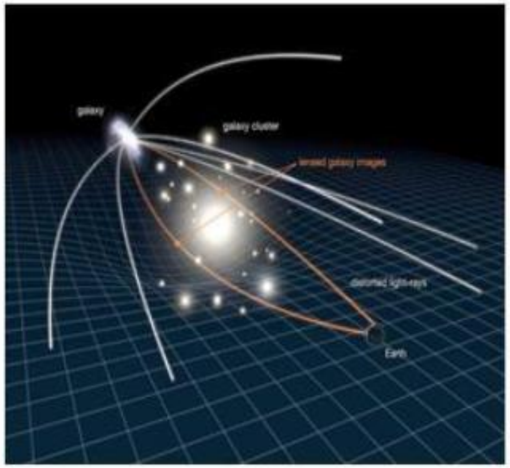
Cloaking



But, can light accelerate?

Any problem with Einstein?

Or with QM?



Gravitational lensing

It can - in media!

Also, in quant mech: Accelerating Airy wave

M.V. Berry and N. L. Balazs, “Nonspreading wave packets, Am. J. Phys. 47, 264 (1979)

Free-particle Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t}+\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}=0$$

Unique Airy wave-packet solution:

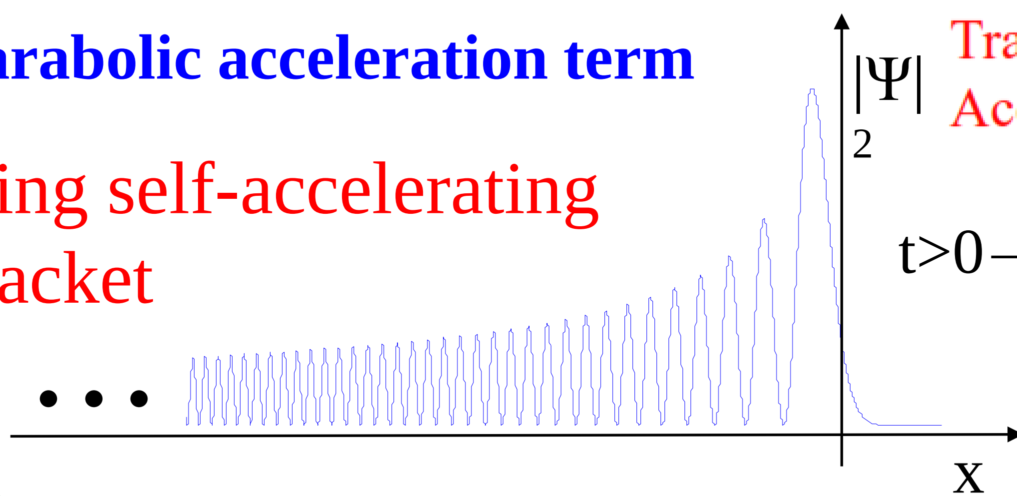
$$\psi(x,t)=\mathrm{Ai}\left[\frac{B}{\hbar^{2/3}}\left(x-\frac{B^3t^2}{4m^2}\right)\right]e^{(iB^3t/2m\hbar)[x-(B^3t^2/6m^2)]}.$$

Parabolic acceleration term

Traveling wave: x - vt

Accelerating: x - 1/2 at^2

Non-diffracting self-accelerating Airy wave-packet



t>0 ⟶ acceleration

Courtesy of Ady Arie

Then, from Quantum Mechanics to Optics:

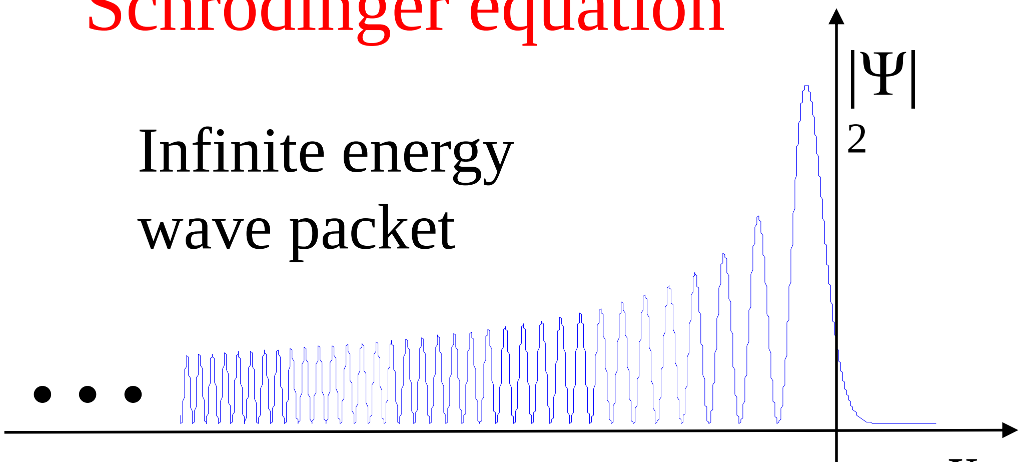
$$i\hbar \frac{\partial \psi}{\partial t} + \frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = 0$$

$$i \frac{\partial \Phi}{\partial \xi} + \frac{1}{2} \frac{\partial^2 \Phi}{\partial s^2} = 0$$

Free particle Schrödinger equation

Scaled paraxial wave equation

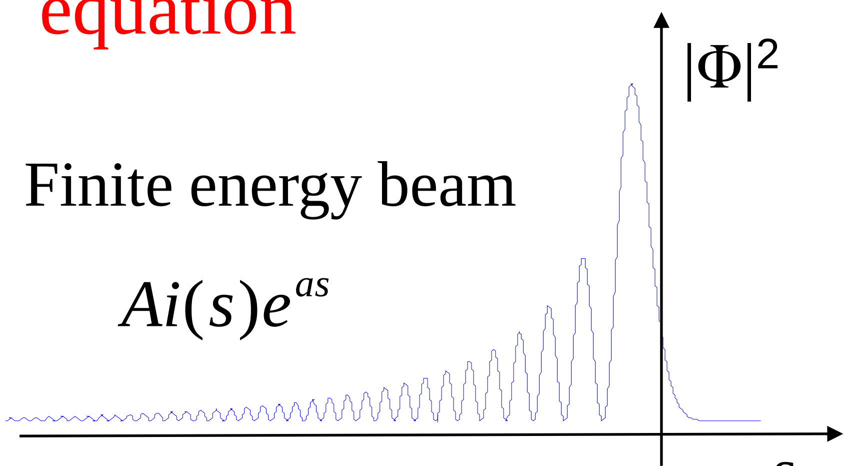
Infinite energy wave packet



Berry and Balzas, 1979

Finite energy beam

$Ai(s)e^{as}$



Siviloglou and Christodulides, 2007

- Non-diffracting Airy
- Freely accelerating

- Nearly non-diffracting
- Freely accelerating

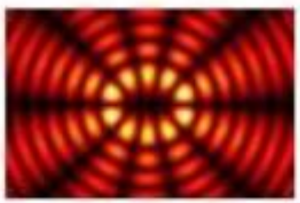
Berry and Balzas, *Am. J. Phys*, **47**, 264 (1979)
Siviloglou, Broky, Dogariu, & Christodoulides, *Phys. Rev. Lett.* **99**, 213901 (2007)

Nondiffracting optical waves

2D

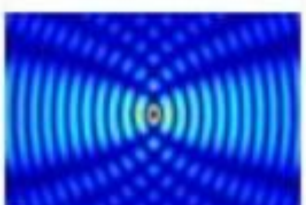
Bessel beam

(Cylindrical coordinate)



Mathieu beam

(Elliptic coordinate)



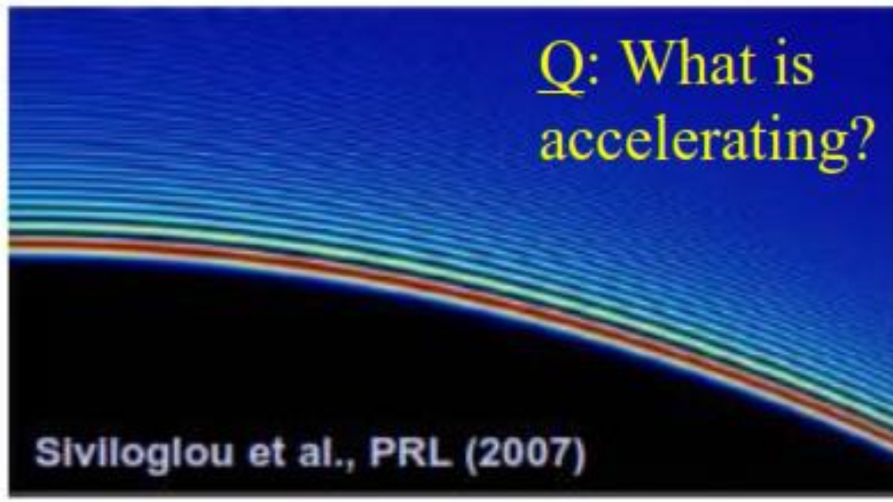
Parabolic beam

(Parabolic coordinate)

Airy beam in 1D

$$\psi(s,\xi) = Ai\left(s-\frac{\xi^2}{4}\right)\exp\left[i\left(\frac{s\xi}{2}-\frac{\xi^3}{12}\right)\right]$$

Free fall



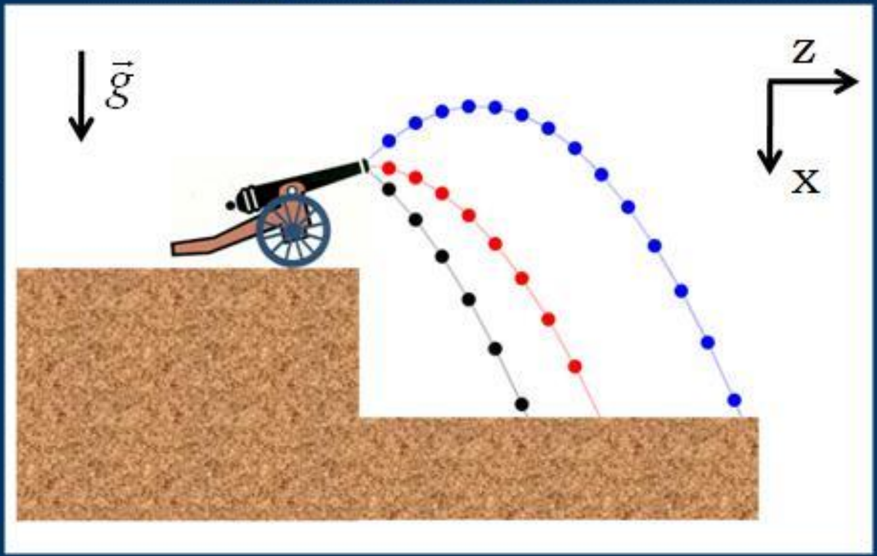
- The only possible nondiffracting wave in 1D
- Self-healing property
- Transverse momentum (self-bending)

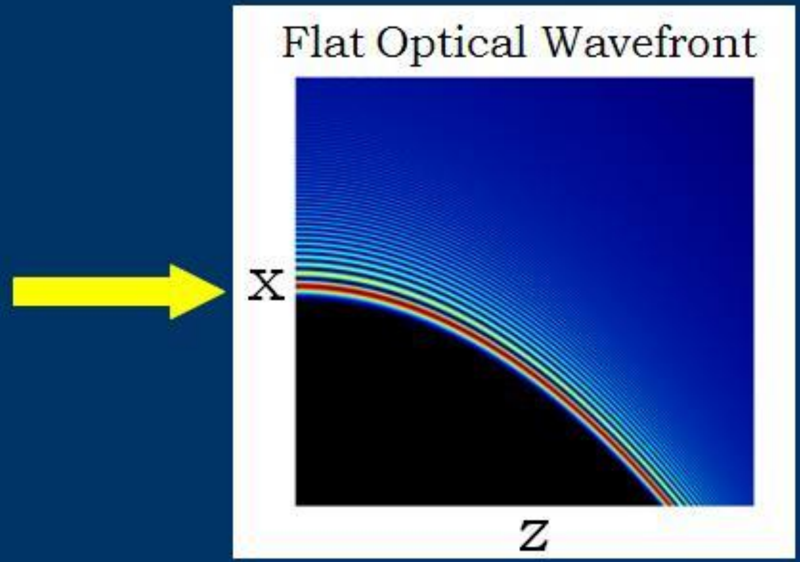
Courtesy of Z. Chen

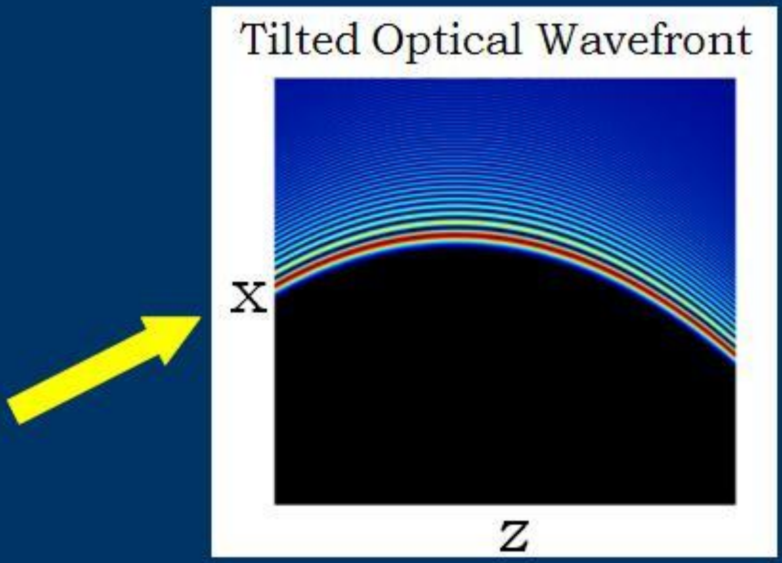
San Francisco State University

Optical analog of projectile ballistics

The Airy beam moves on a parabolic trajectory very much like a projectile under the action of gravity!



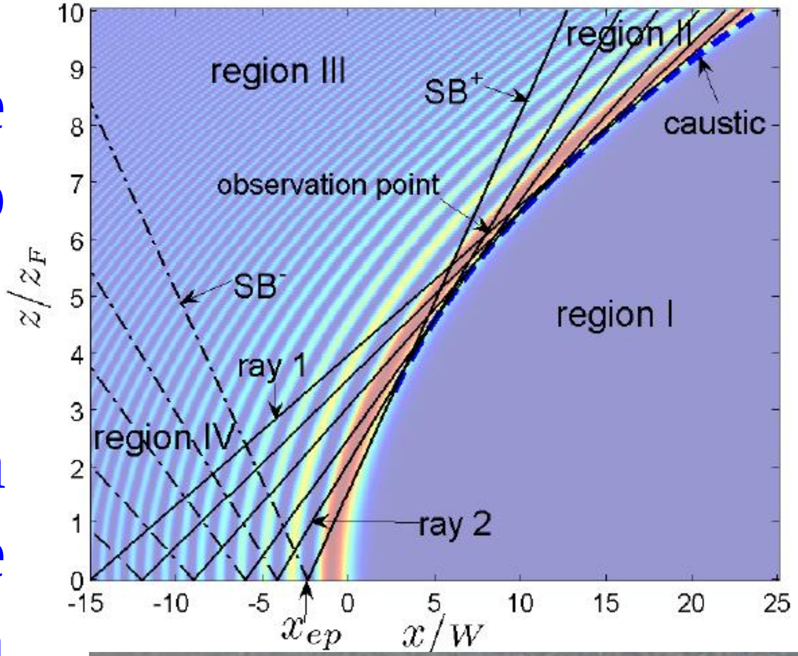




Airy beam – a manifestation of caustic

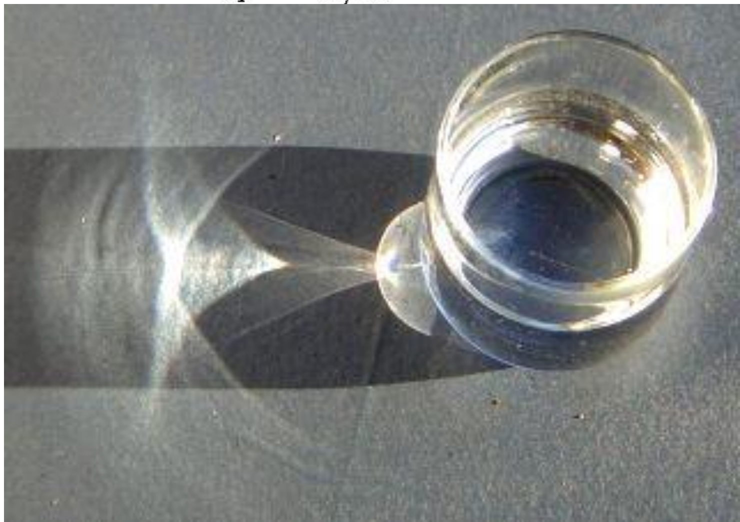
Caustic: An envelope of light rays reflected or refracted by a curved surface or an object, or the projection of that envelope of rays on another surface.

In ray description, the rays are tangent to the parabolic line but do not cross it.



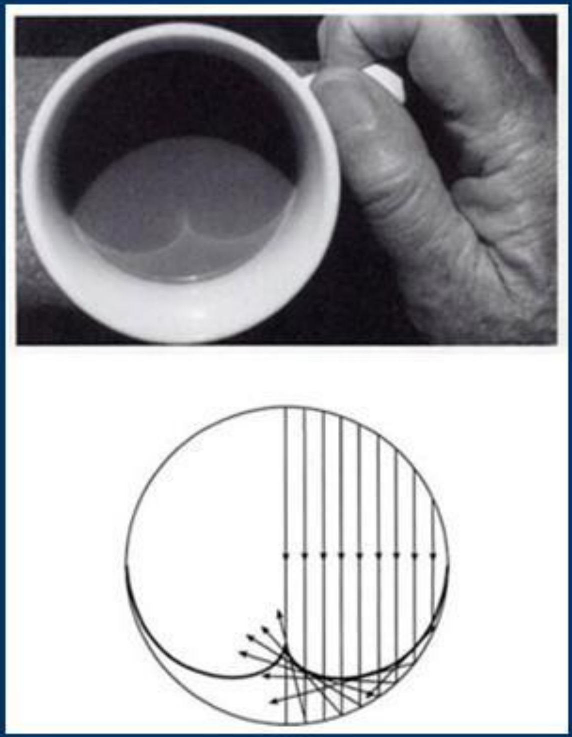
The Airy beam is a beam with curving parabolic trajectory, but the "center of mass" of the beam propagates along a straight line!

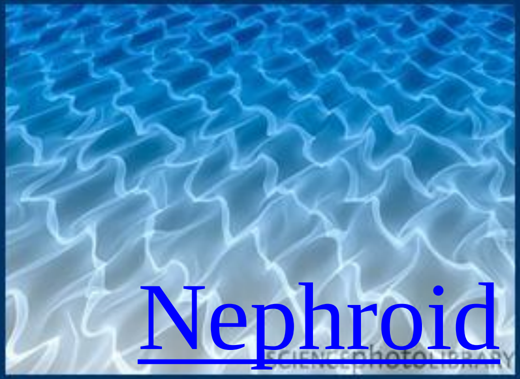
Curved caustics in everyday life



Kaganovsky and Heyman, *Opt. Exp.* **18**, 8440 (2010)

Caustics are everywhere





[Nephroid](#) caustics

Airy beams propagating in NL media

- Equation $$i\frac{\partial\psi}{\partial z} - i\frac{z}{2}\frac{\partial\psi}{\partial x} + \frac{1}{2}\frac{\partial^2\psi}{\partial x^2} + \delta n\psi = 0.$$
- Input beam $$\psi(x) = A_1\mathrm{Ai}[(x-B)]\exp[a(x-B)] + \exp(il\pi)A_2\mathrm{Ai}[-(x+B)]\exp[-a(x+B)],$$

Single

Beam

Prop

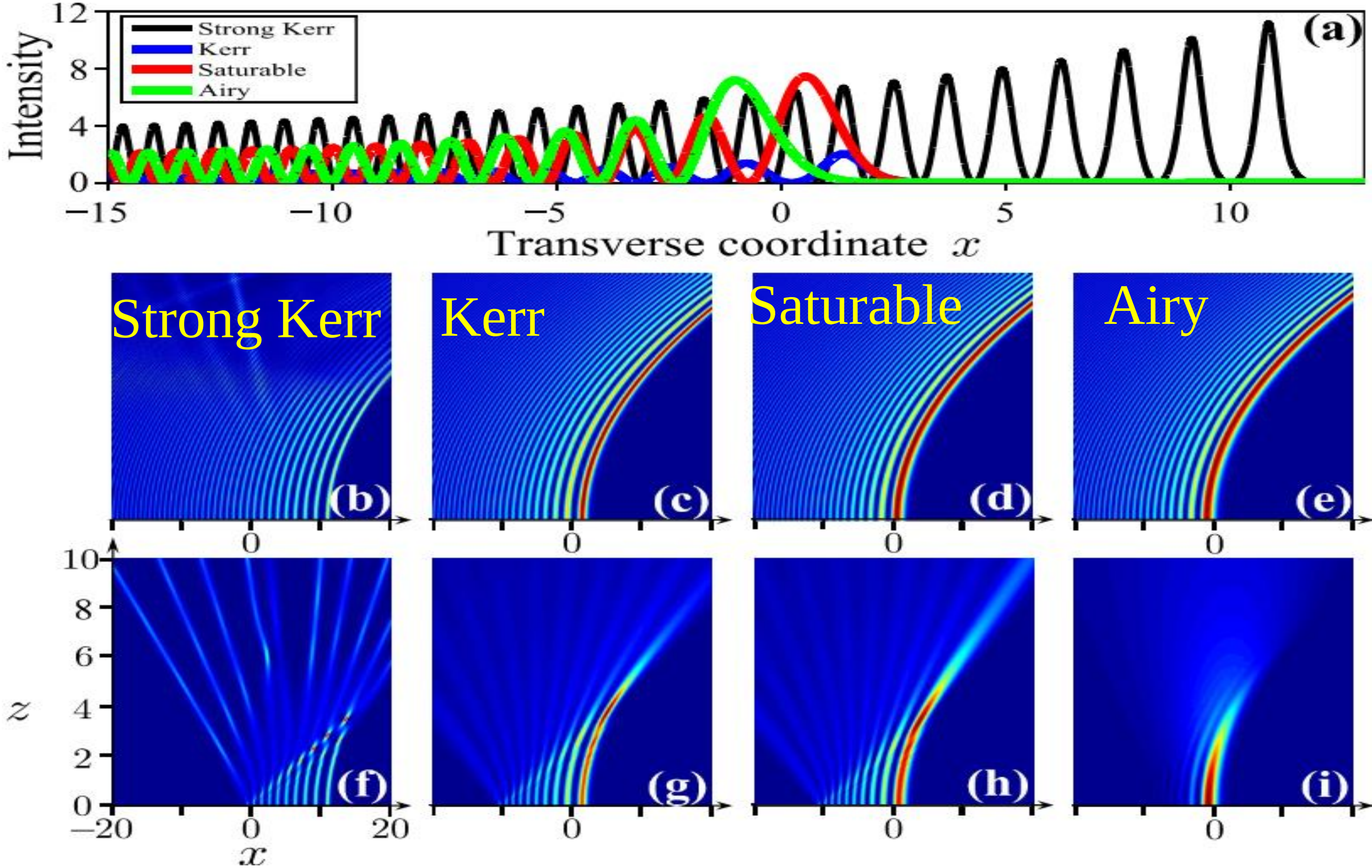
No trunca-

tion, a=0

Truncated

a=0.2

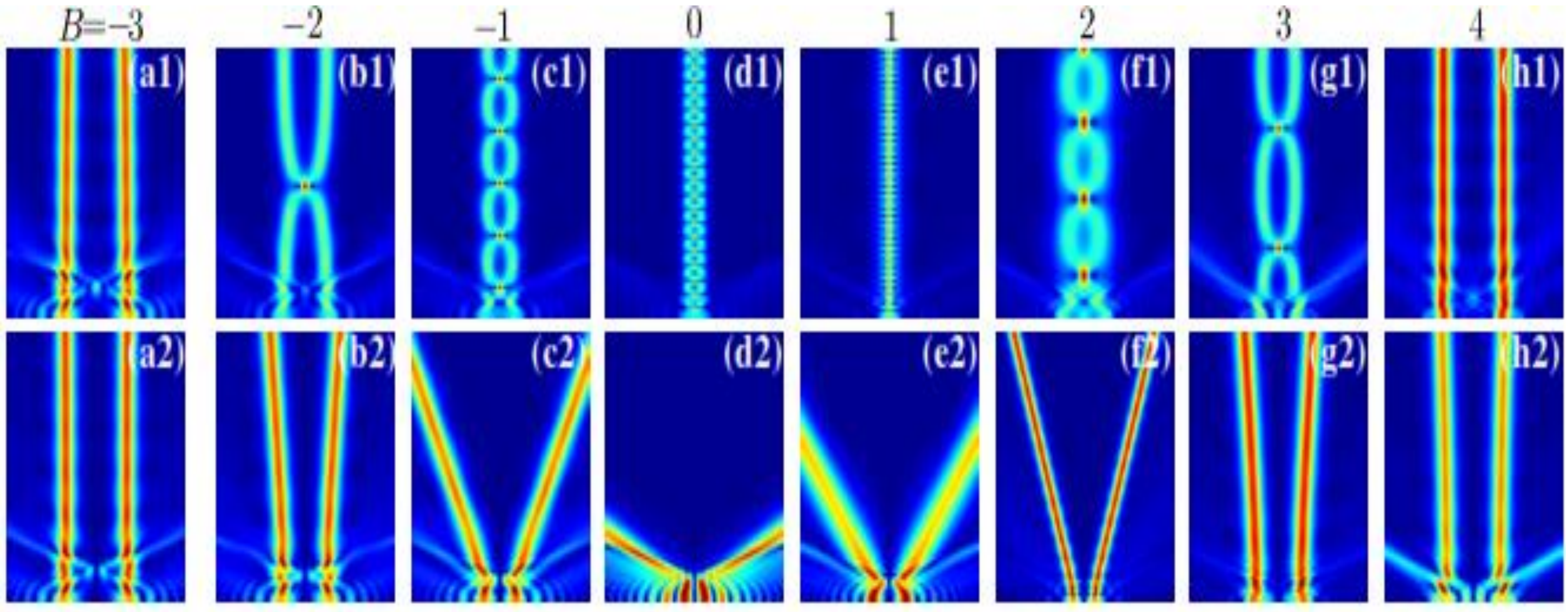
Solitons!



Counter-accelerating Airy beams

- Kerr medium: Generation of solitons
- No acceleration!
- Upper row: In-phase (attraction)
- Lower row: Out-of-phase (repulsion)

Y. Zhang *et al.* *Appl. Sci.* 7, 341 (2017)



Scorer beams: Inhomogeneous cousins of Airy beams

Introduced by Richard Scorer in 1950

Equation

$$i\frac{\partial u}{\partial \xi}+\frac{1}{2}\frac{\partial^2 u}{\partial X^2}+\gamma V(\xi,X)u=0,$$

Presumed solution

$$u(\xi,X)=A(\xi,X)e^{\lambda(\xi,X)a+iB(\xi,X)}$$

Amplitude

$$A(\xi,X)=\frac{F(\theta)}{\sqrt{w_0}},$$

Phase

$$B(\xi,X)=k(\xi)+b(\xi)X+c(\xi)X^2,$$

Decay factor

$$\lambda(\xi,X)=X+\rho\xi^2$$

Potential

$$V(\xi,X)=\frac{1}{2w_0^2F}$$

Scorer diff. eq.

$$\frac{\partial^2 F}{\partial \theta^2}-\theta F=-\frac{1}{\pi},$$

Proper Scorer function

$$F(\theta)=Gi(\theta),\quad Gi(\theta)=\frac{1}{\pi}\int_0^\infty \sin\left(\theta t+\frac{1}{3}t^3\right)dt$$

Recall Airy diff. eq.

$$\frac{\partial^2 F}{\partial \theta^2}-\theta F=0$$

Zhong *et al.*, PLA 528 (2024) 130023

A primer on Airy and Scorer functions

Airy diff equation

$$\frac{d^2 w}{d z^2} - z w = 0.$$

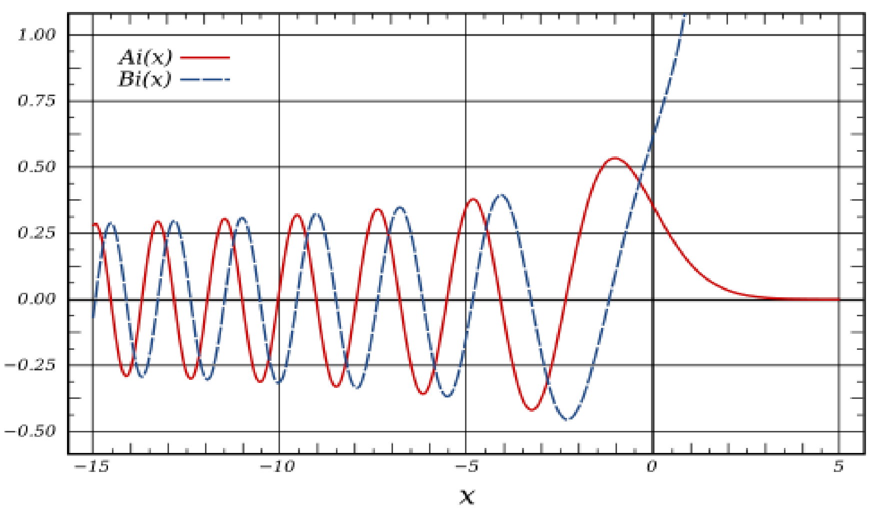
Solutions

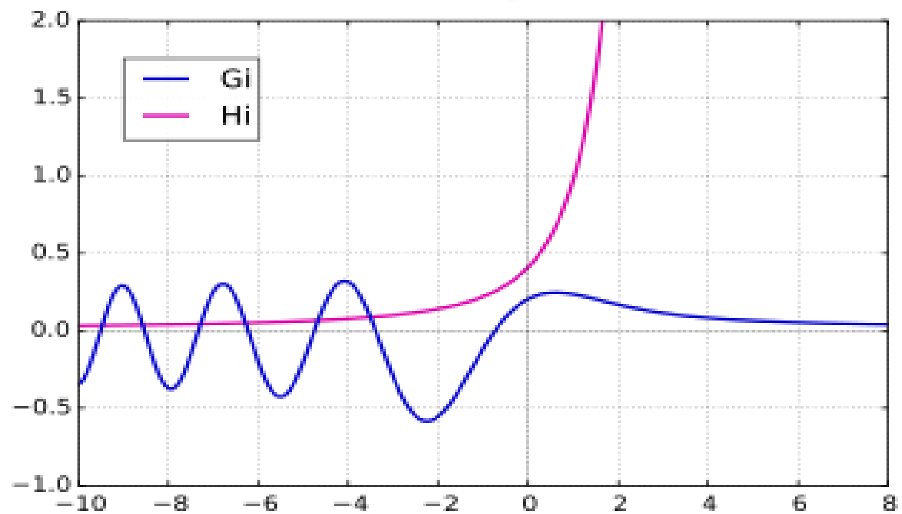
$$\mathrm{Ai}(z) = \frac{1}{\pi} \int_0^\infty \cos\left(zt + \tfrac{1}{3}t^3\right)\, dt,$$

$$\mathrm{Bi}(z) = \frac{1}{\pi} \int_0^\infty \sin\left(zt + \tfrac{1}{3}t^3\right)\, dt + \frac{1}{\pi} \int_0^\infty e^{zt - \frac{1}{3}t^3}\, dt$$

Scorer diff equations $w'' - z\,w = -1/\pi \qquad w'' - z\,w = 1/\pi$

$$\mathrm{Gi}(z) = \frac{1}{\pi} \int_0^\infty \sin\left(zt + \frac{1}{3}t^3\right)\, dt \qquad \mathrm{Hi}(z) = \frac{1}{\pi} \int_0^\infty e^{zt - \frac{1}{3}t^3}\, dt.$$



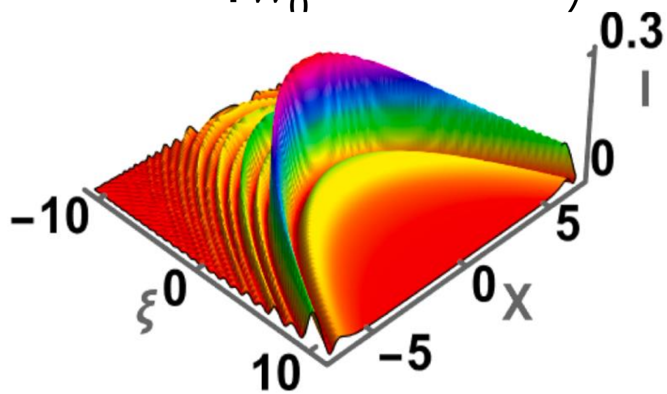


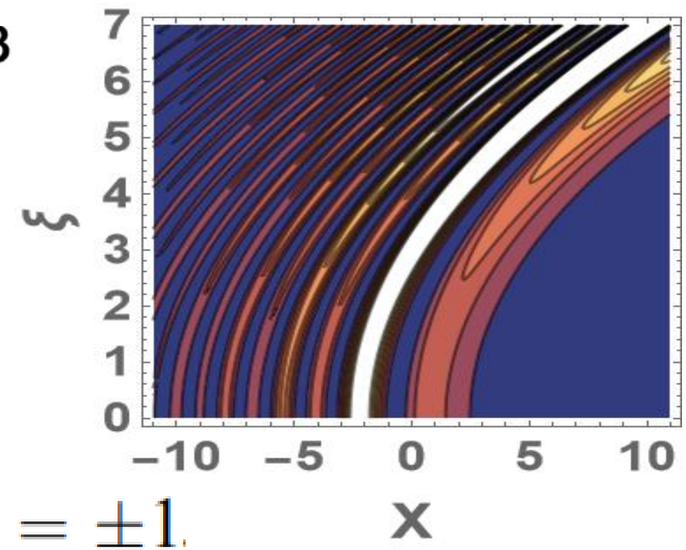
Counterpropagating Scorer beams

Scorer beam

$$u(\xi, X) = \frac{1}{\sqrt{w_0}} Gi\left(\frac{4w_0^3 X - \xi^2 + 4w_0^3 a i \xi}{4w_0^4}\right) e^{aX - \frac{a}{2w_0^3}\xi^2 + i\left(-\frac{\xi^3}{12w_0^6} - \frac{\xi^2}{8w_0^3} + \frac{a^2\xi}{2} + \frac{\xi}{2w_0^3}X\right)}$$

Zhong *et al.*
Comm. Th. Phys. 77 (2025) 055501





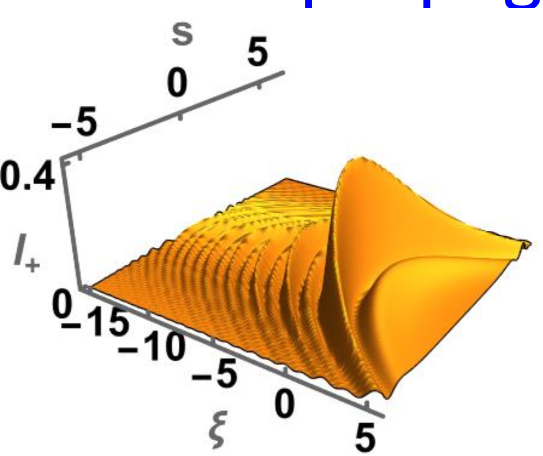
Input beams:

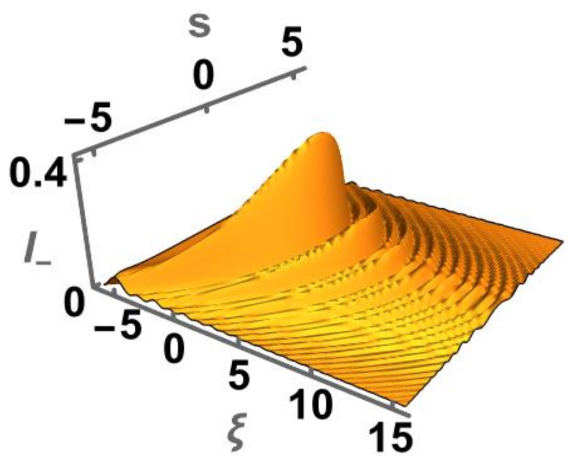
$$u(\xi = 0, s) = \frac{1}{\sqrt{w_0}} \mathrm{Gi}\left(\frac{\sigma s}{w_0}\right) \mathrm{e}^{\sigma\lambda s}$$

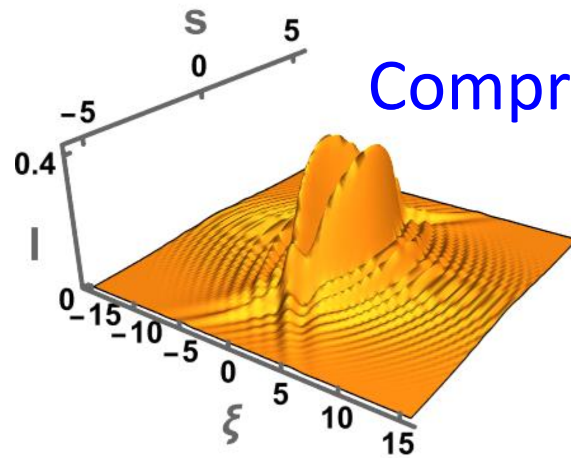
Direction:

$\sigma = \pm 1$

Counterpropagating beams







Compressed Beam

Controlled self-bending of Scorer beams

Equation

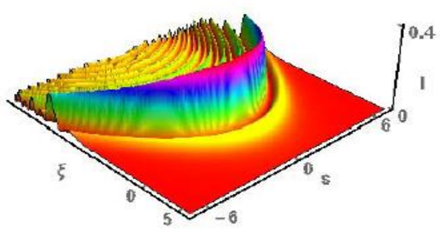
$$i\frac{\partial u}{\partial \xi}+\frac{1}{2}\frac{\partial^2 u}{\partial s^2}+qsu+\gamma V(\xi,s)u=0$$

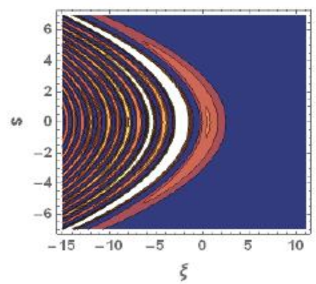
$$u(\zeta=0,s)=Gi(s)e^{\alpha s}$$

One-parameter self-similar solution

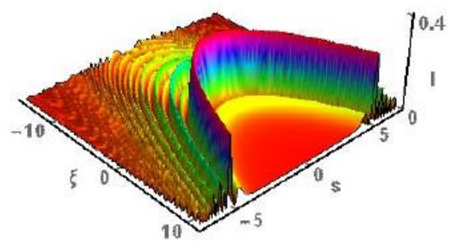
$$u(\xi,s)=Gi\left(s-\frac{1+2q}{4}\xi^2+\alpha i\xi\right)e^{\alpha s-\frac{1+q}{2}\alpha\xi^2+i\left(-\frac{1+3q+2q^2}{12}\xi^3-\frac{\xi^2}{8}+\frac{1}{2}\alpha^2\xi+\frac{1+2q}{2}\xi s\right)}$$

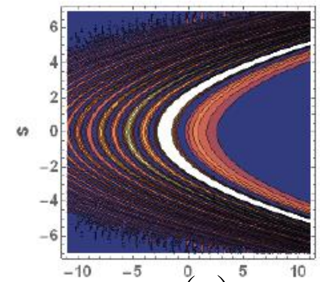
Positive q



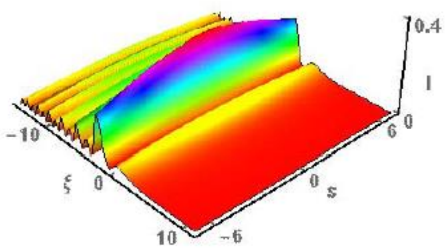


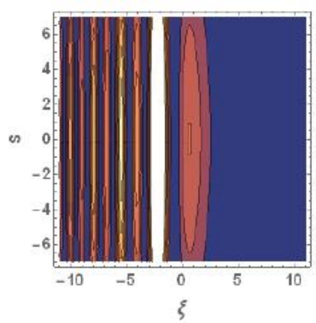
Negative q





Compensated, q=-1/2



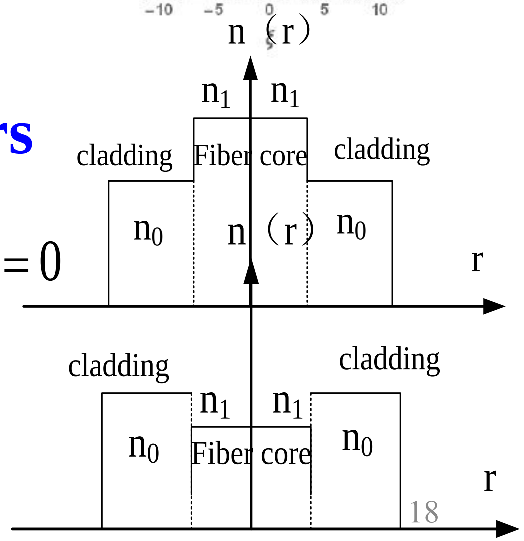


Application: Fibers

$$i\frac{\partial u}{\partial z}+\frac{1}{2}\frac{\partial^2 u}{\partial x^2}+\frac{k\Delta n}{n_0}xu+\gamma Vu=0$$

$\Delta n = n_1 - n_0 > 0 \qquad q > 0$

$\Delta n = n_1 - n_0 < 0 \qquad q < 0$



Yang et al., Self-bending Scorer beams

Scorer beams in highly nonlocal media

Snyder-Mitchel model of accessible solitons

NL nonlocal equation

$$i\frac{\partial u}{\partial \xi}+\frac{1}{2}\frac{\partial^2 u}{\partial s^2}+\chi(\xi)N(I)(\xi,s)+\gamma V(\xi,s)u=0.$$

Snyder-Mitchel linear eq.

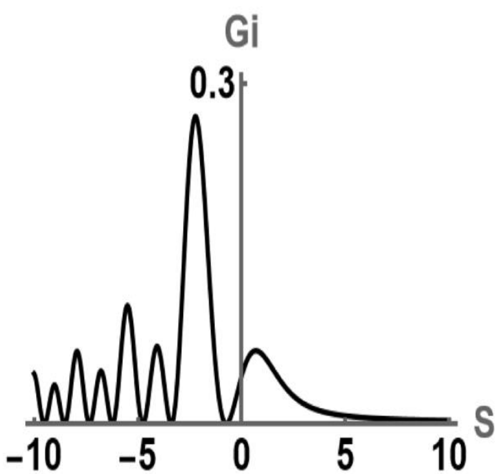
$$i\frac{\partial u}{\partial \xi}+\frac{1}{2}\frac{\partial^2 u}{\partial s^2}-\chi(\xi)s^2u+\gamma V(\xi,s)u=0$$

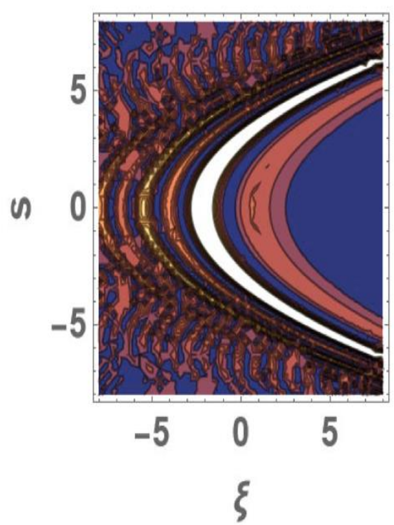
Accessible Scorer soliton

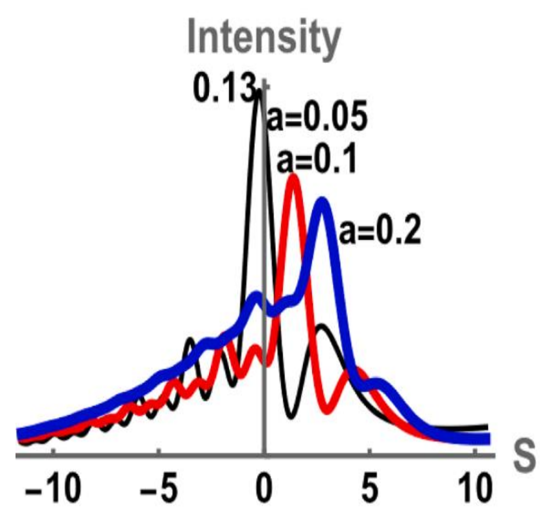
$$u(\xi,s)=\frac{1}{\sqrt{w_0}}Gi\left(\frac{4w_0^3s-\xi^2}{4w_0^4}\right)e^{i\left[-\frac{1}{12w_0^6}\xi^3-\frac{\xi^2}{8w_0^3}+\frac{1}{2w_0^3}\xi s\right]}$$

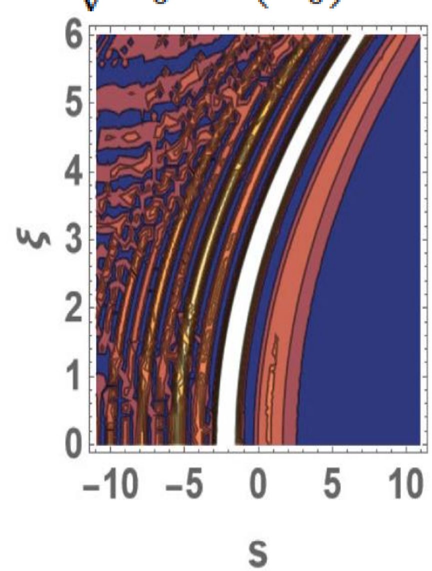
Decay factor

$$u(\xi=0,s)=\frac{1}{\sqrt{w_0}}Gi\left(\frac{s}{w_0}\right)e^{as}$$









Zhong *et al.*, Wave Motion 132 (2025) 103442

Summary

Introduced interacting Airy and Scorer beams

In linear media: Superposition and interference; no solitons

In NL-NL media: NL interference and generation of solitons

Depicted counter-accelerating Scorer beams in Kerr media

Demonstrated one-parameter bending family of Scorer beams

Presented accessible solitons based on Scorer beams

Thank you for your attention!